# (ASTRO)FIZIKA U UČIONICI: DA ILI NE

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Deo aktivnosti AD Alfa u 2022/23. godini realizuju se u okviru projekta „Kako dohvatiti zvezde“, uz finansijsku podršku Centra za promociju nauke

# KAKO OVO ISKORISTITI U UČIONICI? ☺

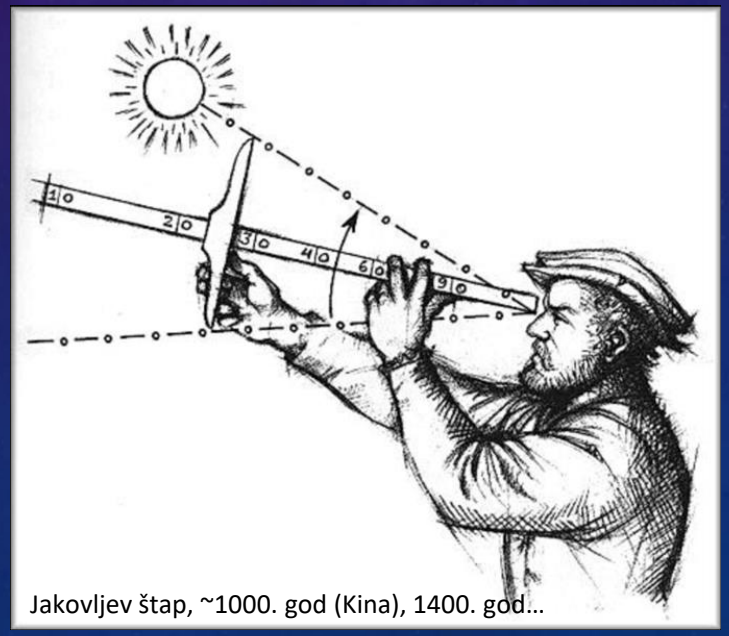
- „Istorija“ astronomije
  - Npr. merenje rastojanja, obim Zemlje, rotacija Sunca itd
  - Neki jednostavni (astro)fizički fenomeni
  - Stare tehnike – lako razumeju, simuliraju, realizuju…

- Savremena astronomija
  - Mnogo (većina?) podataka je dostupna „free“
  - Može za učenje, ali koristi se i za pisanje radova… ☺

# „ISTORIJA“ ASTRONOMIJE

Sekstant, 1760 i 1800+…

Kvadrant, 1450 (možda i 1200. god.)



Jakovljev štap, ~1000. god (Kina), 1400. god…

# „ISTORIJA“ ASTRONOMIJE

# KORISNI ALATI

- Program „Stellarium“, https://stellarium.org/
  - Pozicija miša na ekranu? ☺ Na primer: https://sourceforge.net/projects/mpos/
- CLEA vežbe, http://public.gettysburg.edu/~marschal/clea/CLEAhome.html
- SOHO fotografije, https://soho.nascom.nasa.gov/
- HelioViewer https://www.helioviewer.org/
- Astronomy Education at University of Nebraska – Lincoln, https://astro.unl.edu/

- Sloan Digital Sky Survey (SDSS), https://www.sdss.org/

# NEKI PRIMERI

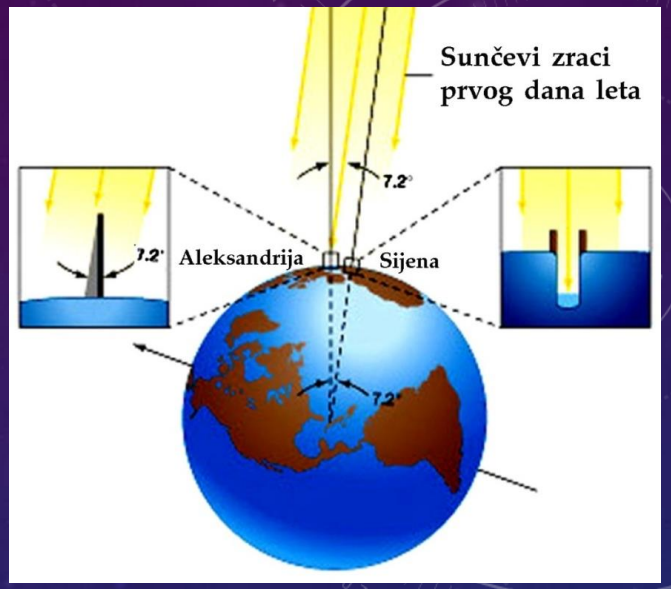
- Merenje obima Zemlje
- Merenje rastojanja do planeta Sunčevog sistema
- Merenje brzine svetlosti
- Određivanje mase Jupitera / Saturna
- Merenje astronomske jedinice
- Merenje perioda rotacije Sunca
- Određivanje mase supermasivne crne rupe
- Snimanje tranzita ekstrasolarnih planeta

# OBIM ZEMLJE

# KAKO „IZMERITI“ OBIM ZEMLJE?

- Koristeći materijal dostupan u učionici izmeriti obim Zemlje ☺

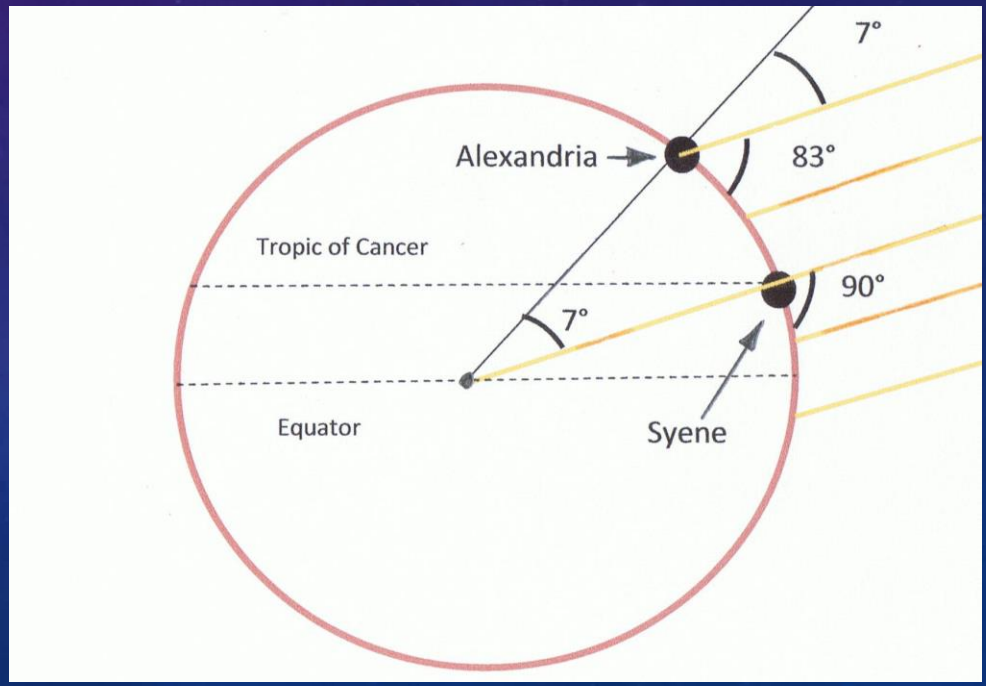
# ERATOSTEN IZ KIRENE:




- Odredio obim Zemlje, na osnovu toga što je uočio da u isto vreme u godini u Aleksandriji Sunčevi zraci padaju normalno na površinu, a u Sijeni pod uglom od oko 7$^{o}$ (oko 50. deo kruga) u odnosu na normalu.
- Zaključio da je to zbog toga što je Zemlja okrugla.
- Utvrdio je da je rastojanje između ova dva grada oko 5000 stadija, i da je obim Zemlje oko 252 000 stadija.
- Smatra se da 1 stadij odgovara dužini oko 185 m.
- Procenjuje se da su njegove procene vrednosti obima Zemlje između 36.690 i 46.620 km.
- Danas utvrđena vrednost je 40.008 km.
- Njegova procena rastojanja Zemlja-Mesec bila je 780.000 stadija (oko 144.000 km), što je znatno manje od stvarne prosečne vrednosti (384.400 km).
- Eratostenova procena rastojanja Zemlja-Sunce bila je vrlo bliska današnjoj (804.000.000 stadija ili oko 148,7 miliona km).

# OBIM ZEMLJE

- Eratosten (276 p.n.e. - 194 p.n.e.)
- Sijena – prvi dan leta, u podne, Sunce tačno iznad
- Aleksandrija – senka!
- Razlika – 7°
- Rastojanje 7 / 360°

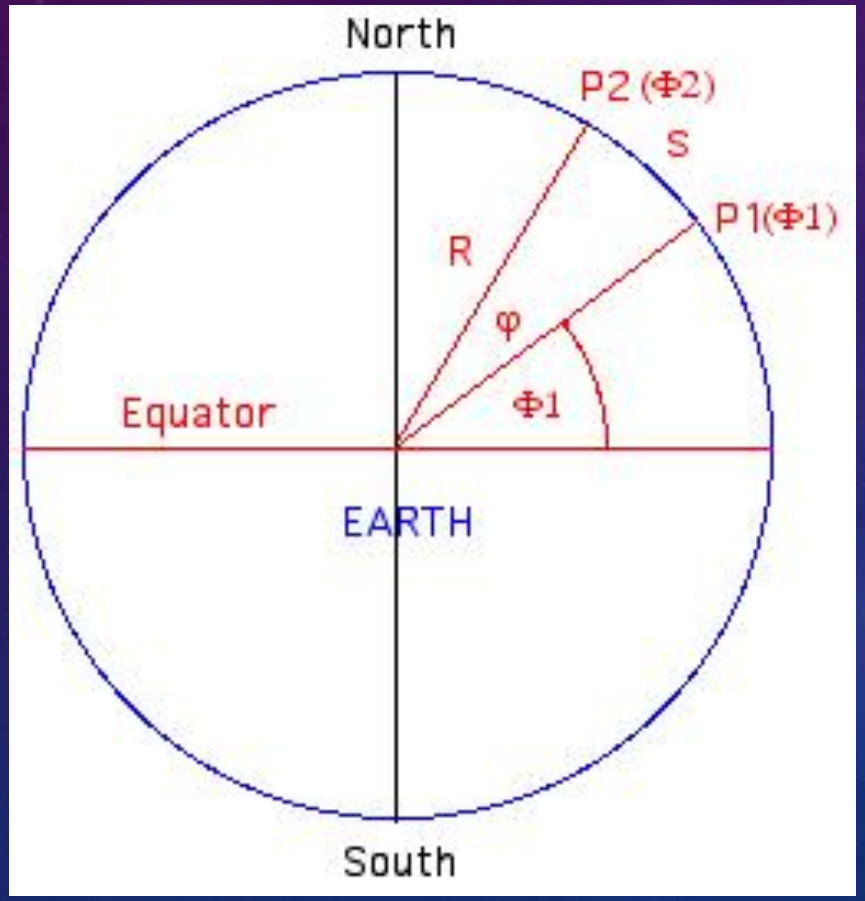


# REŠENJE


SAMSUNG
12:45 PM
12:45 PM
Thu, March 14
New York
Sunny 59°F
Google
Email
Calendar
Camera
Play Store
Phone
Contacts
Messaging
Internet
Apps

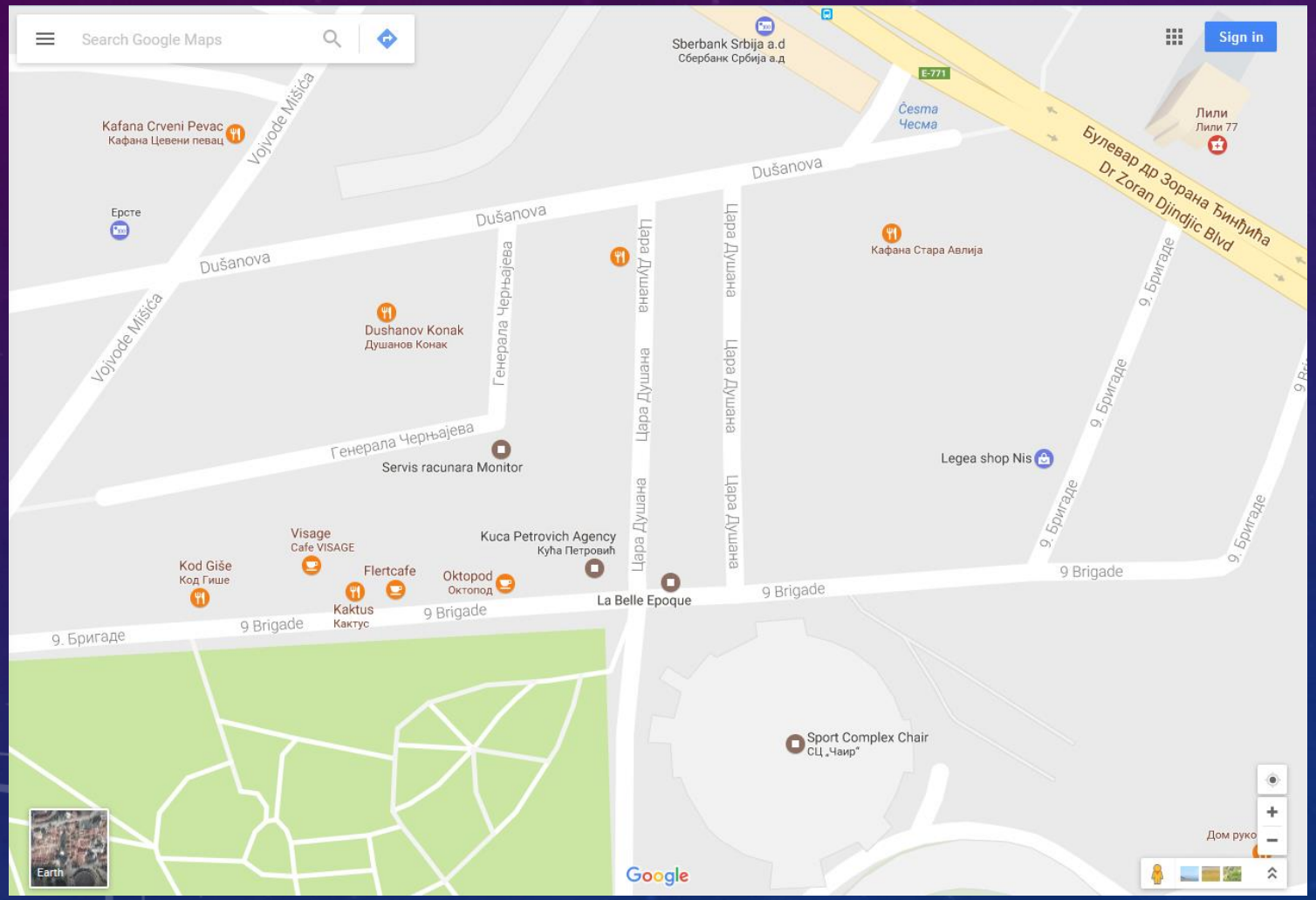

Stellarium
stellarium.org

# OBIM ZEMLJE I MOBILNI TELEFON




$$\frac{\varphi}{s} = \frac{360^\circ}{2\pi R} \qquad L = 2\pi R = \frac{s \cdot 360^\circ}{\varphi}$$

# ILI GOOGLE MAP ☺




$$\phi_1 = 43{,}317254$$
$$\phi_2 = 43{,}318517$$

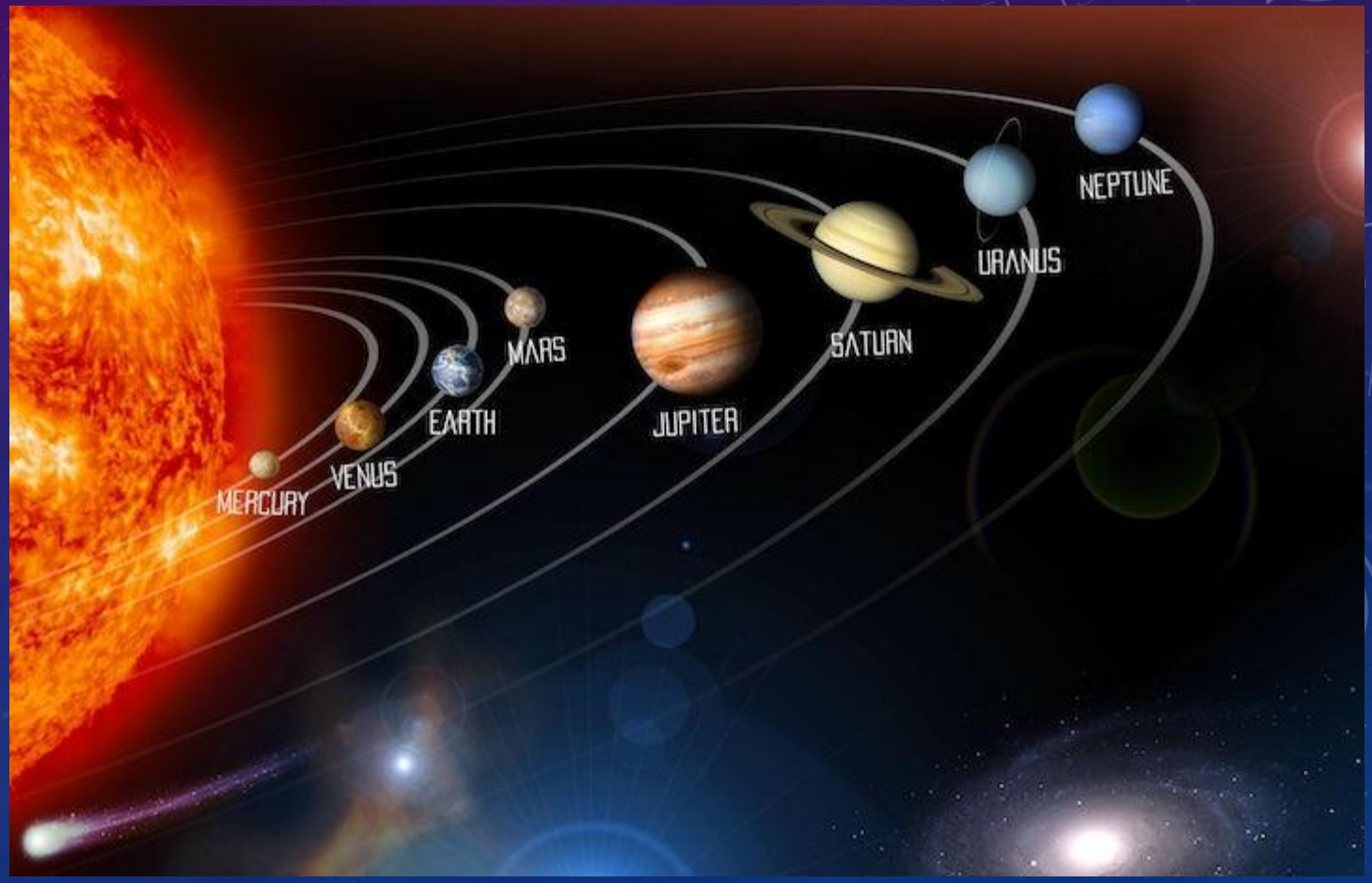
$$\Delta\phi = \left|\phi_1 - \phi_2\right| = 0{,}001263$$
$$s = 150\text{m}$$

$$L = 2\pi R = \frac{360^o \cdot s}{\phi} \approx 42.755{,}3\text{km}$$
$$R = 6.800\text{km}$$

# MERENJE RASTOJANJA U SUNČEVOM SISTEMU

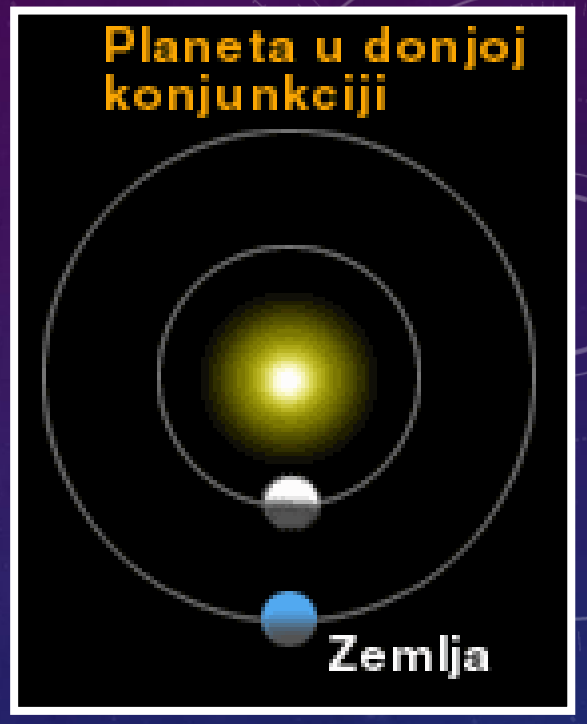
# HELIOCENTRIČNI SISTEM

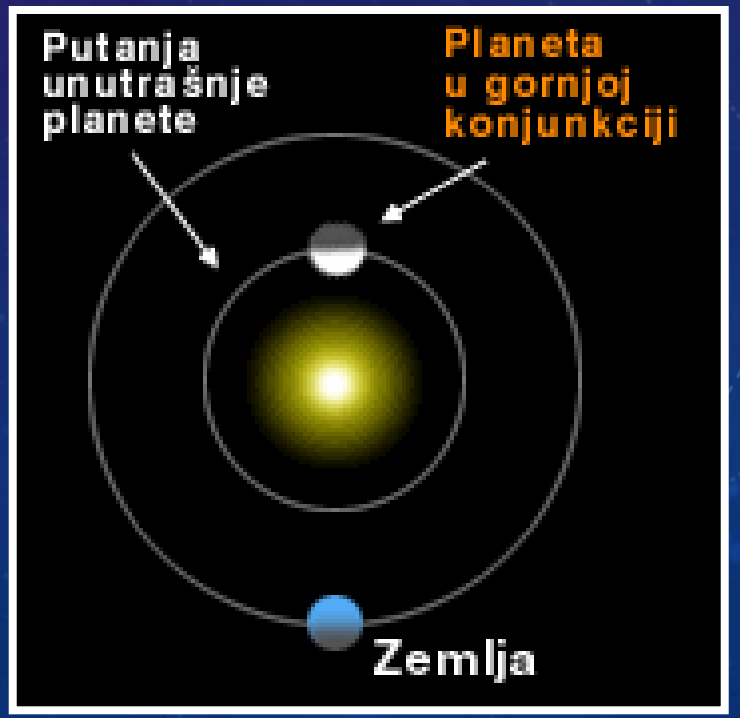
- Postoji više različitih podela:
- „Astrofizički“:
  - planete Zemljinog (terestričke)
  - Jupiterovog (jovijanske) tipa.
- „Astronomski“:
  - **unutrašnje** (donje)
  - **spoljašnje** (gornje).



# KARAKTERISTIČNI POLOŽAJI PLANETA

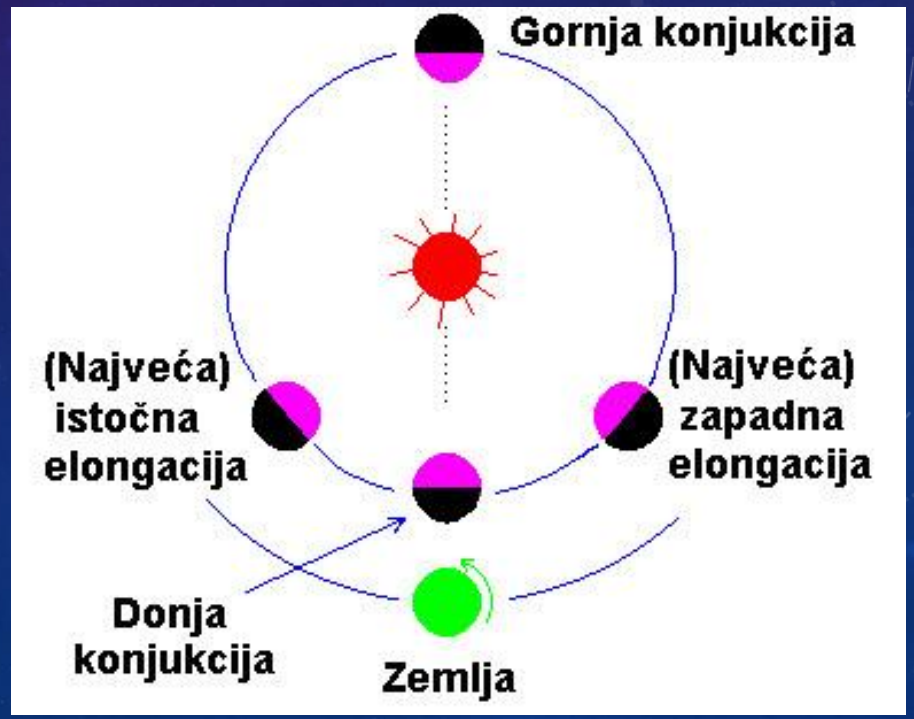
- Unutrašnje planete: gornja i donja konjunkcija
  - Planeta se ne vidi sa Zemlje
  - Gornja - njen sjaj slabiji od Sunčevog,
  - Donja - prema Zemlji okrenuta neosvetljena strana  planete.



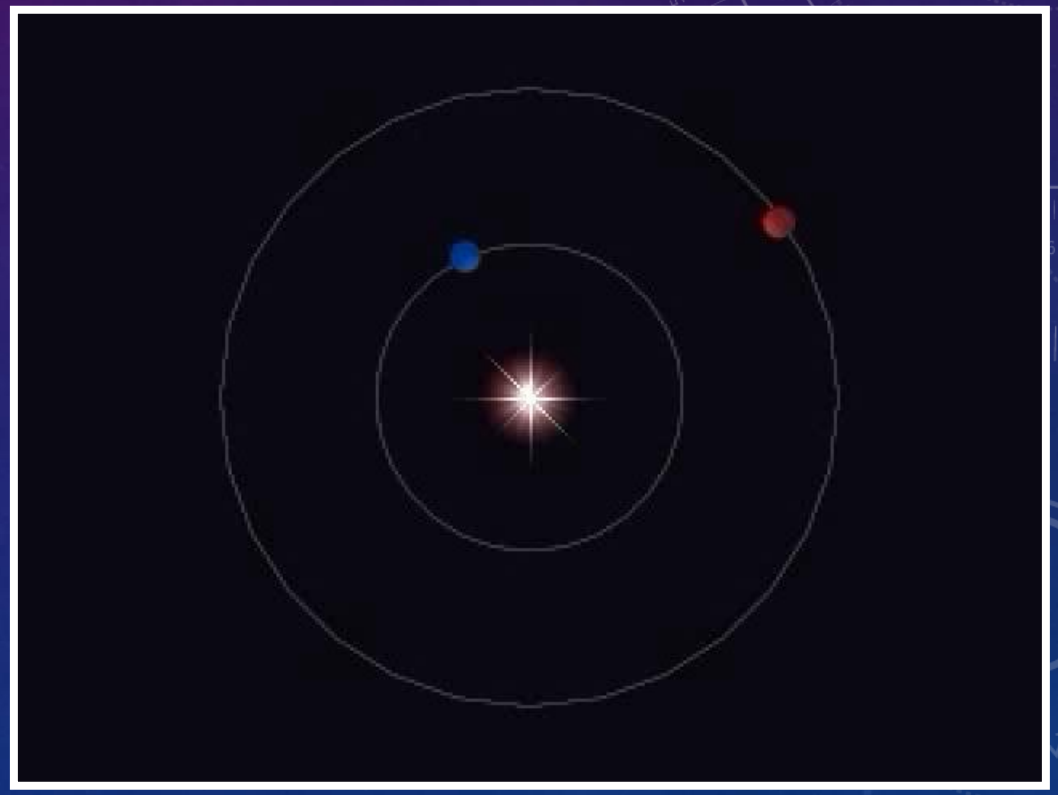
# KARAKTERISTIČNI POLOŽAJI PLANETA

- Unutrašnje planete
- Najbolje vide - za posmatrača sa Zemlje prividno najdalje od Sunca.
  - To su položaji najveće istočne i zapadne elongacije
    - **Istočna** elongacija - **posle zalaska** Sunca,
    - **Zapadna** elongacija - **pre izlaska** Sunca.



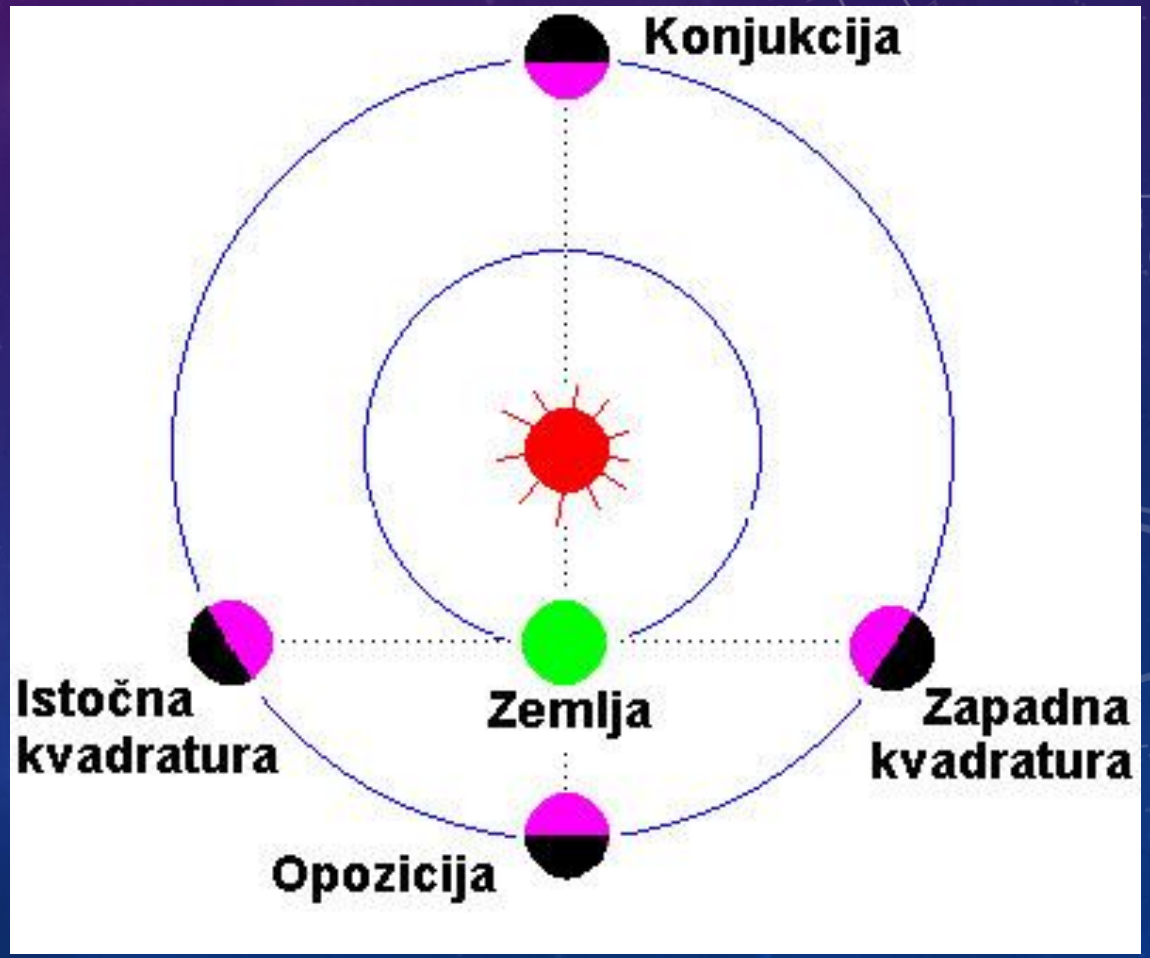
# KARAKTERISTIČNI POLOŽAJI PLANETA

- Spoljašnje planete **najbolje se vide** u položaju kada su planeta, Zemlja i Sunce na jednoj pravoj i kada je spoljašnja planeta nasuprot Sunca, najbliže Zemlji - **opozicija**
- Kulminira u ponoć i vidi se cele noći.



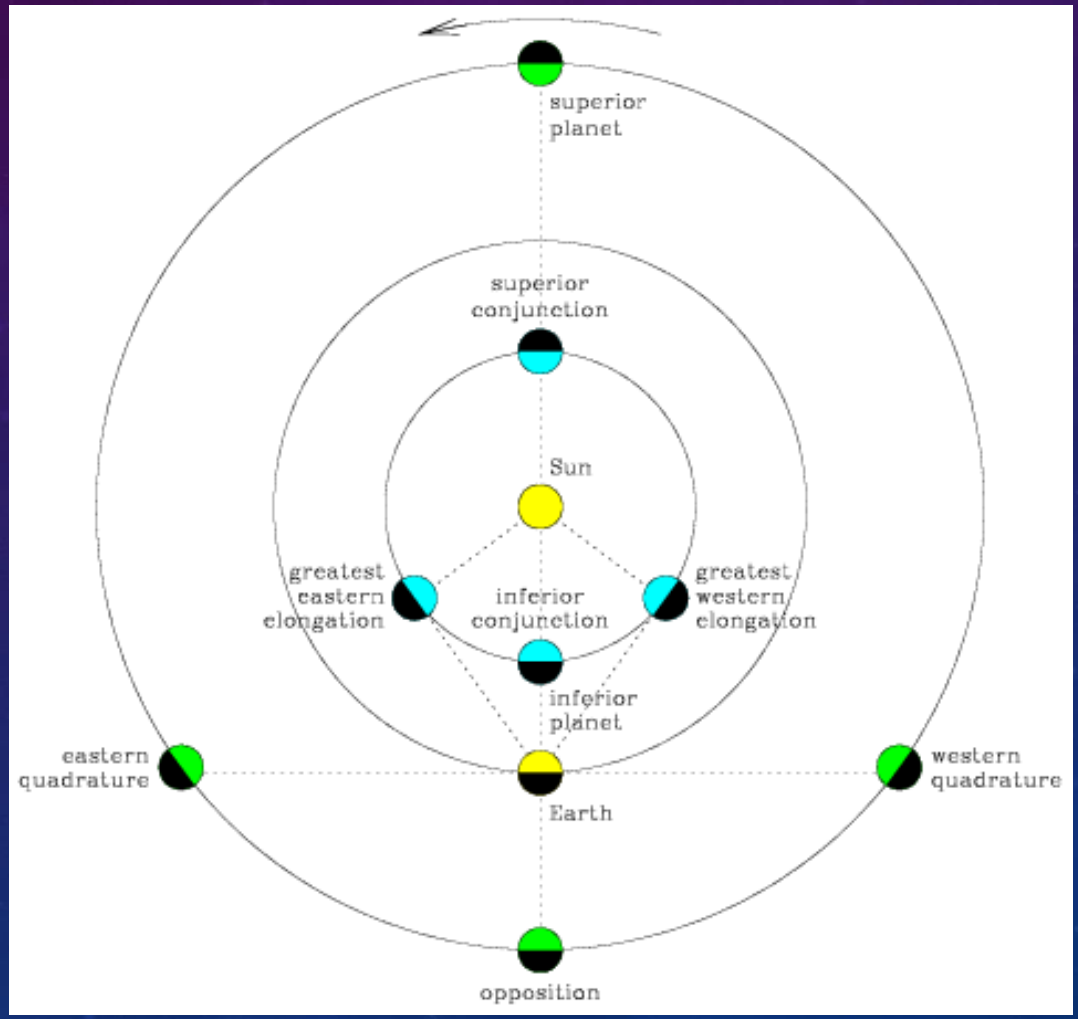
# KARAKTERISTIČNI POLOŽAJI PLANETA

- Sunce - između planete i Zemlje – konjunkcijia
- Karakteristični položaji su i istočna i zapadna kvadratura.
- Ugao koji zaklapaju pravci vizura planeta-Zemlja i Zemlja-Sunce je prav.




Demonstracija: NAAP Labs, *Solar System Models / Planetary Configurations Simulation*

# KARAKTERISTIČNI POLOŽAJI PLANETA

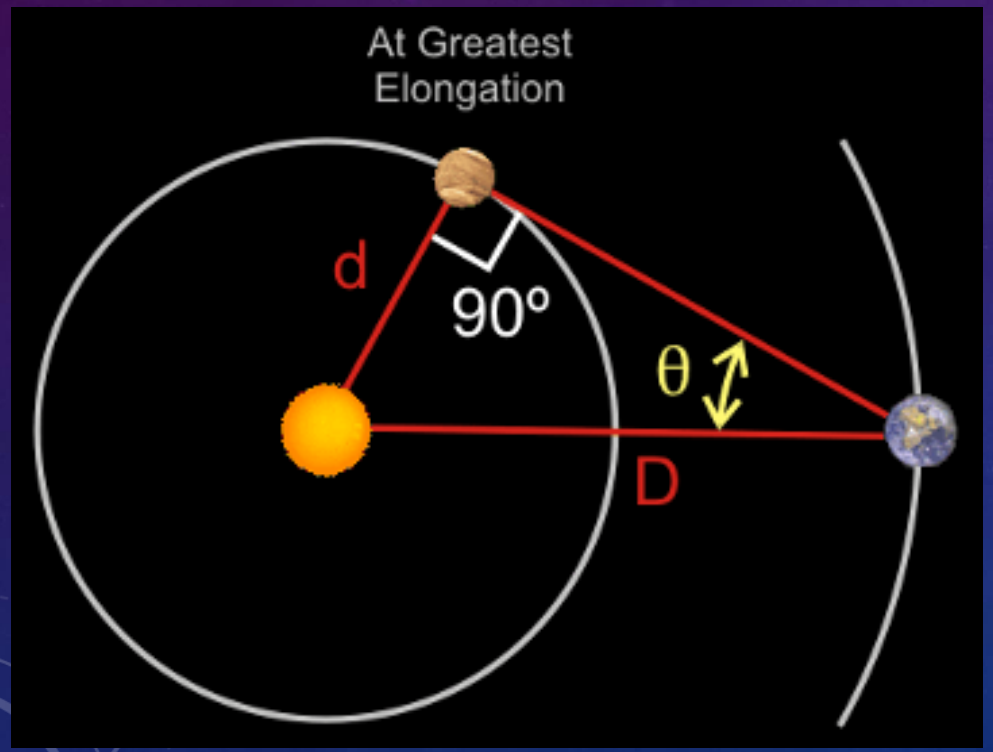


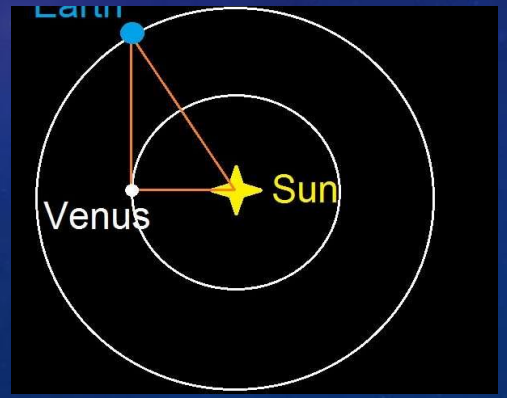
# UNUTRAŠNJE PLANETE

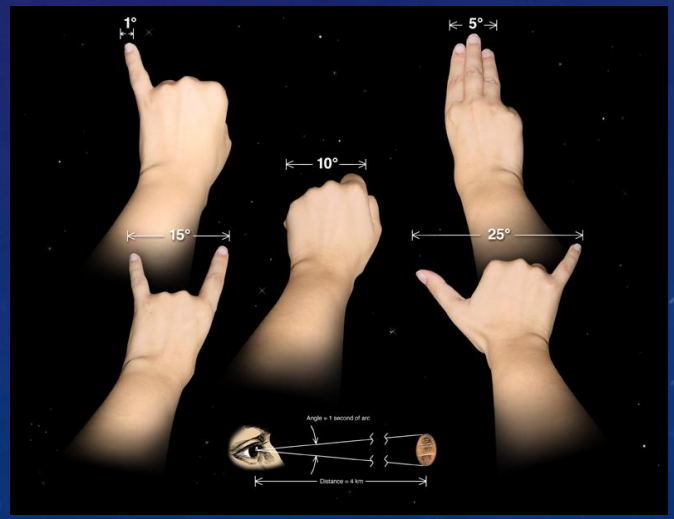
$$d = D \sin\theta$$

$D = 1$ astronomska jedinica

$$d = \sin\theta$$

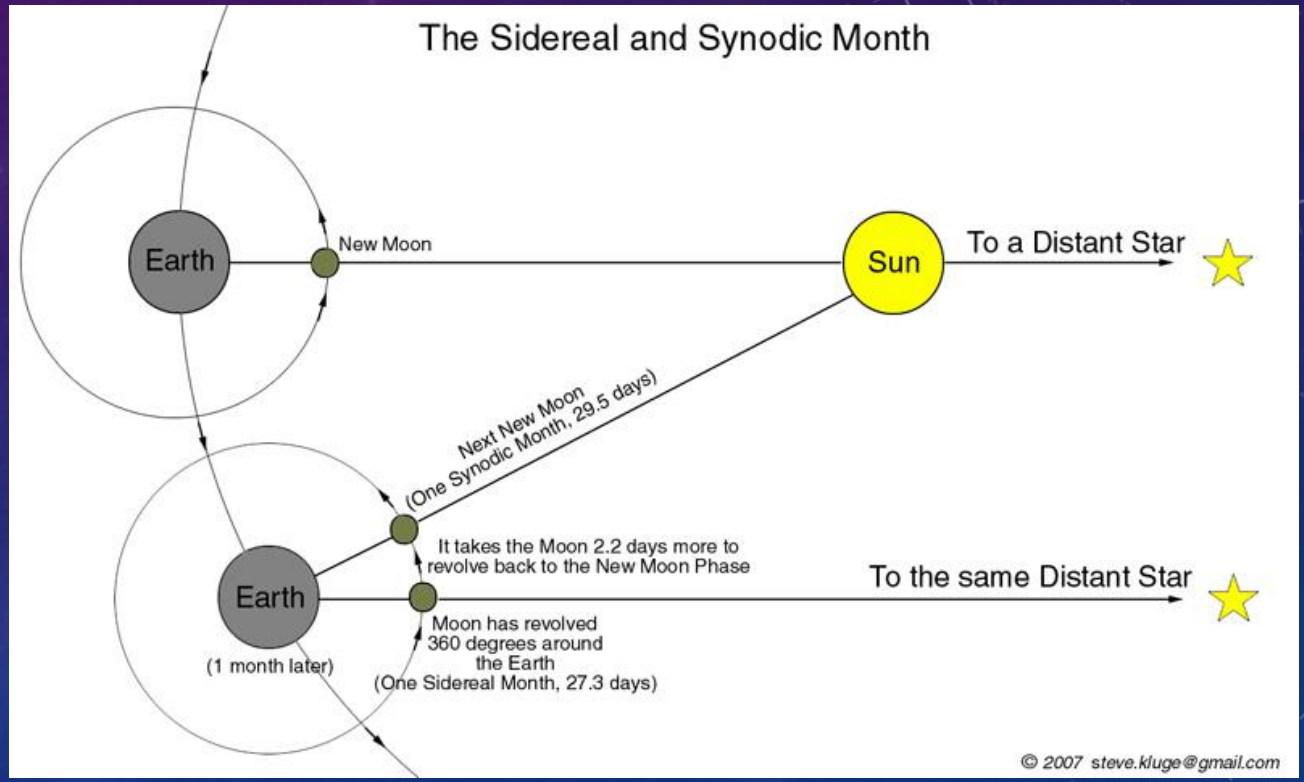


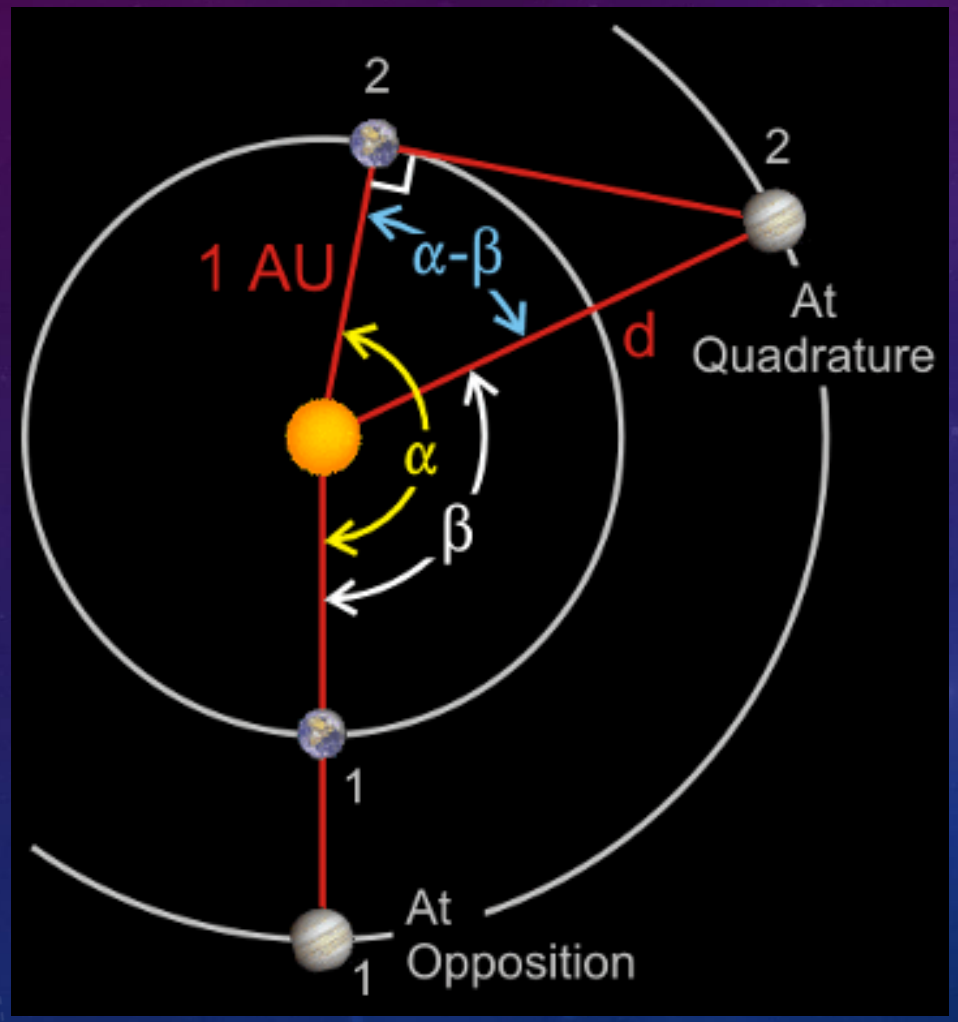



# SIDERIČKI I SINODIČKI PERIOD

- Naći vezu između sinodičkog i sideričkog perioda evolucije.
  - **Siderički** – u odnosu na zvezde
  - **Sinodički** – ono što vidimo (od grčke reči *sastanak*) – vreme između dve identične konfiguracije



# SPOLJAŠNJE PLANETE



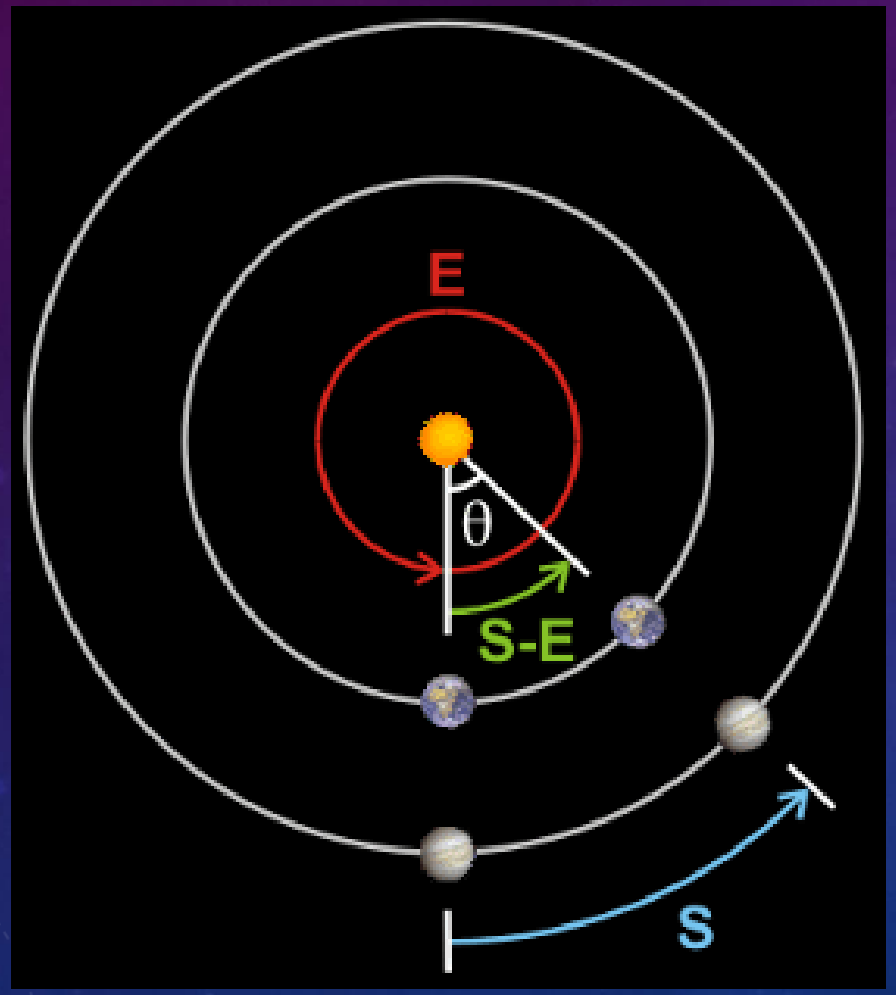

$$\alpha = T\frac{360^o}{E}$$

$$\beta = T\frac{360^o}{P}$$

$$d = \frac{1}{\cos(\alpha - \beta)}$$

*E* – siderički period Zemlje
*P* – Siderički period planete

# SIDERIČKI I SINODIČKI PERIOD




Za vreme *S* planeta pređe ugao $\theta$ a Zemlja $360^o + \theta$

Izračunao Nikola Kopernik

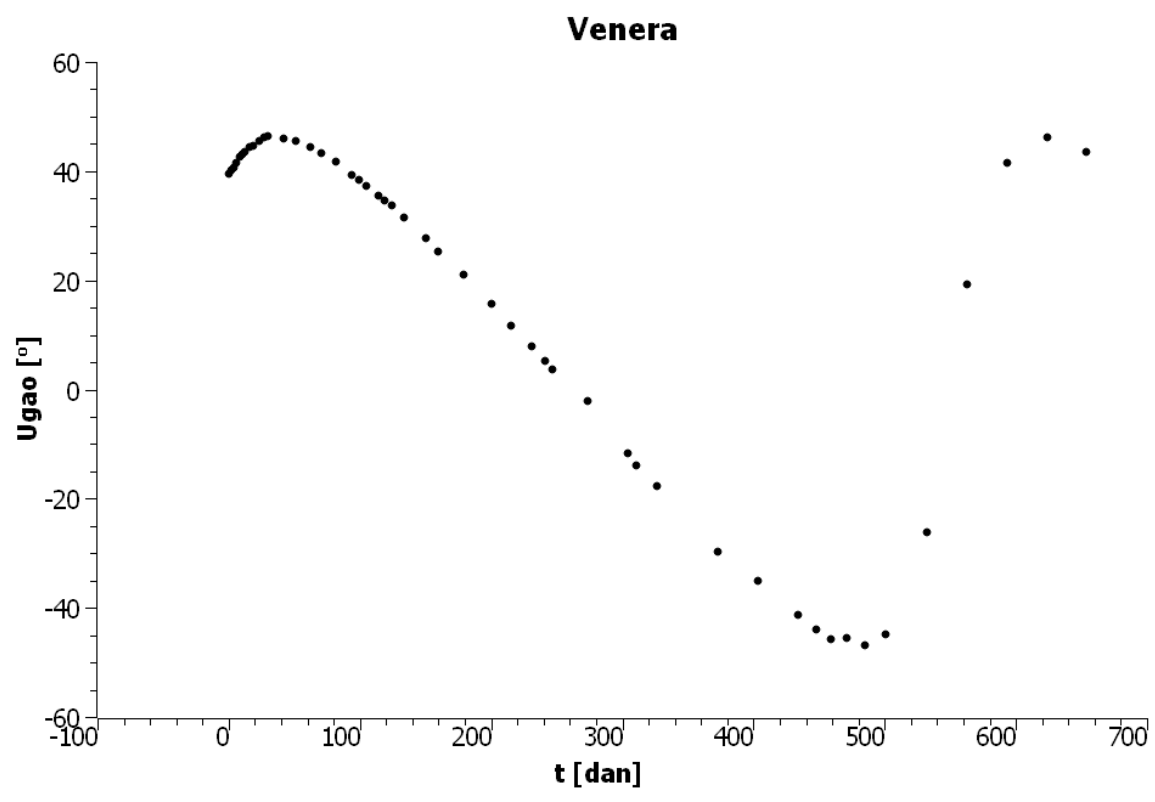
$$\theta = (S - E)\frac{360^o}{E} = S\frac{360^o}{P}$$

$$\frac{1}{S} = \frac{1}{E} - \frac{1}{P}$$ - spoljašnje planete

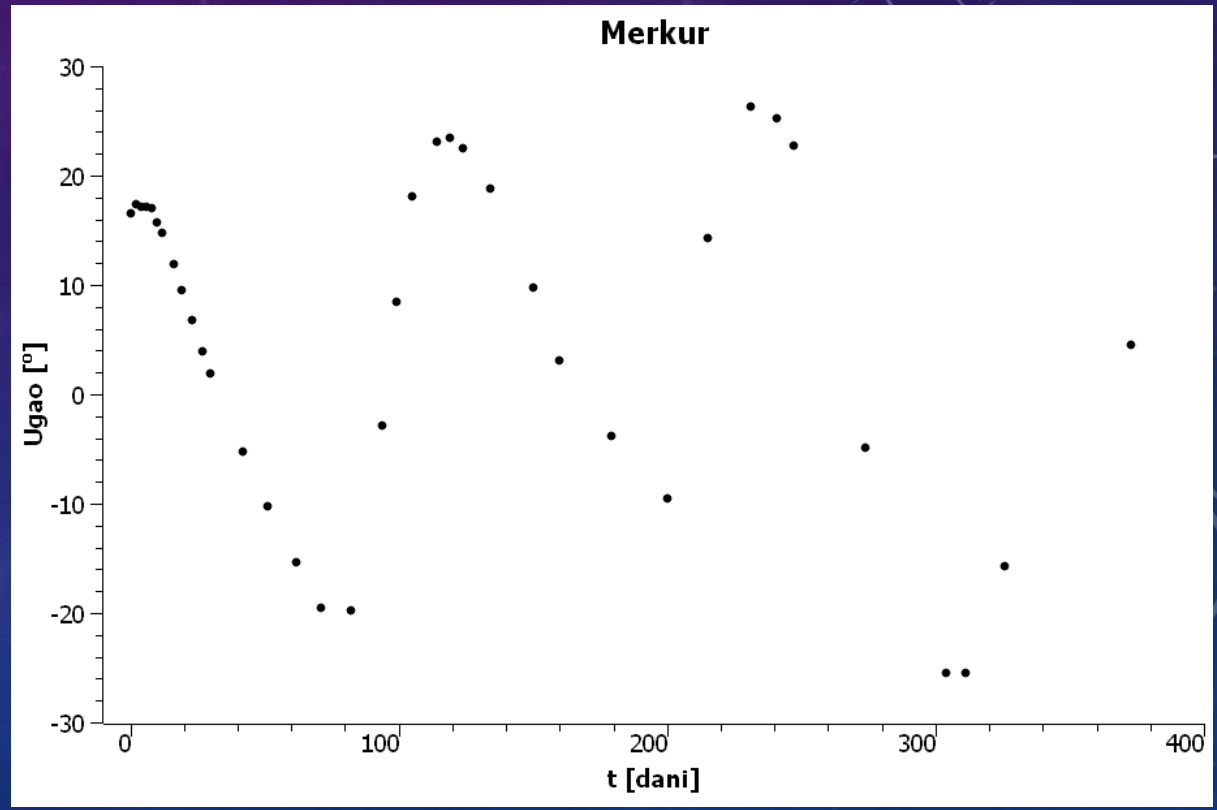
$$\frac{1}{S} = \frac{1}{P} - \frac{1}{E}$$ - unutrašnje planete

*E* – siderički period Zemlje
*P* – siderički period planete
S – sinodički period planete;
vreme za koje planeta pređe ugao $\theta$

# MERKUR I VENERA




$\theta \approx 45^o$ $d_{Venera} = 0.71\ AU$




$\theta \approx 24^o$ $d_{Merkur} = 0.41\ AU$

# GALILEJEVI SATELITI

MERENJE BRZINE SVETLOSTI. 3. KELEROV ZAKON.

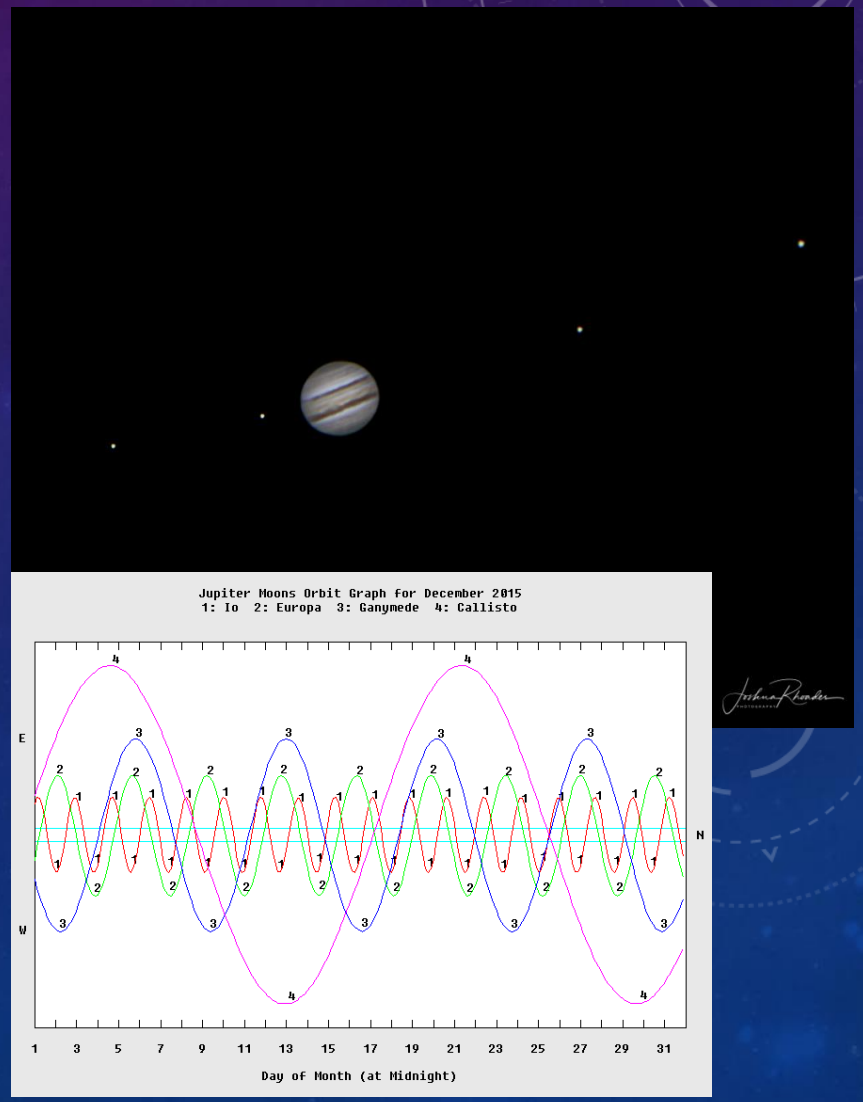
# PRVO MERENJE BRZINE SVETLOSTI

- Ole Remer, 1676. godine
  - Merenje vremena pomračenja Jupiterovih satelita
- Dobijena vrednost $c = 2{,}14 \cdot 10^8\ m/s$
- Tačna vrednost $c = 3{,}00 \cdot 10^8\ m/s$
- Značaj merenja – brzina svetlosti je konačna!

- Pre 300 godina, bez savremene tehnologije
  - Samo 60 godina nakon Galilejevog teleskopa (i otkrića Jupiterovih satelita)

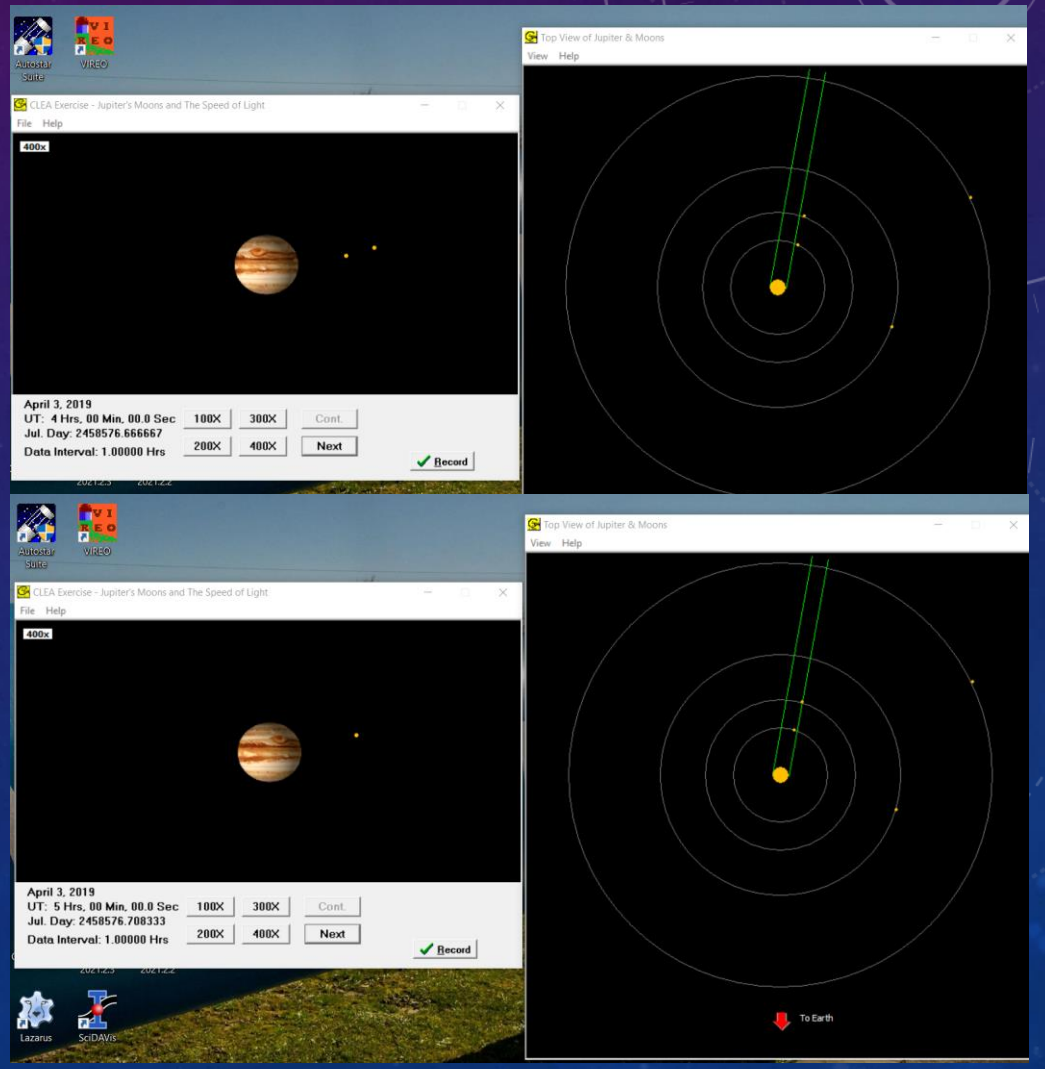
# REMEROVO MERENJE BRZINE SVETLOSTI

- Nije nameravao da izmeri brzinu svetlosti!
- **Cilj:**
  - da li Jupiterovi sateliti mogu da se iskoriste za određivanje geografske dužine (za navigaciju brodova)
- Jupiter kao sat
  - Četiri satelita otkrivena 1610. godine
  - Periodi: Io (1,8 dana), Evropa (3,5 dana), Ganimed (7 dana), Kalisto (16,7 dana)
  - Ako je period pravilan:
    - položaj satelita može da se predvidi za bilo koji trenutak u budućnosti
    - Efemeride



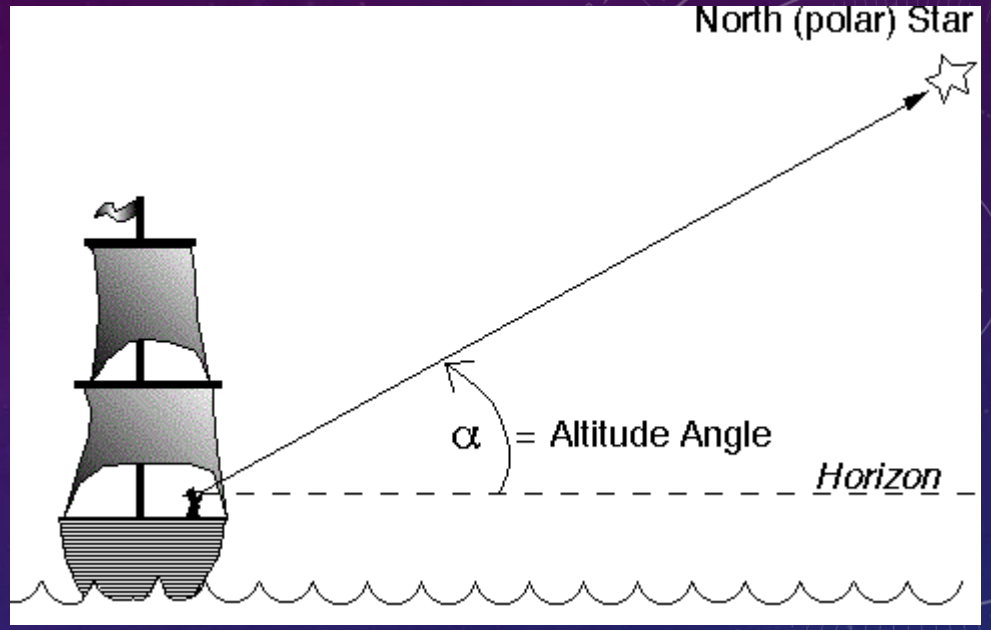
# REMEROVO MERENJE BRZINE SVETLOSTI

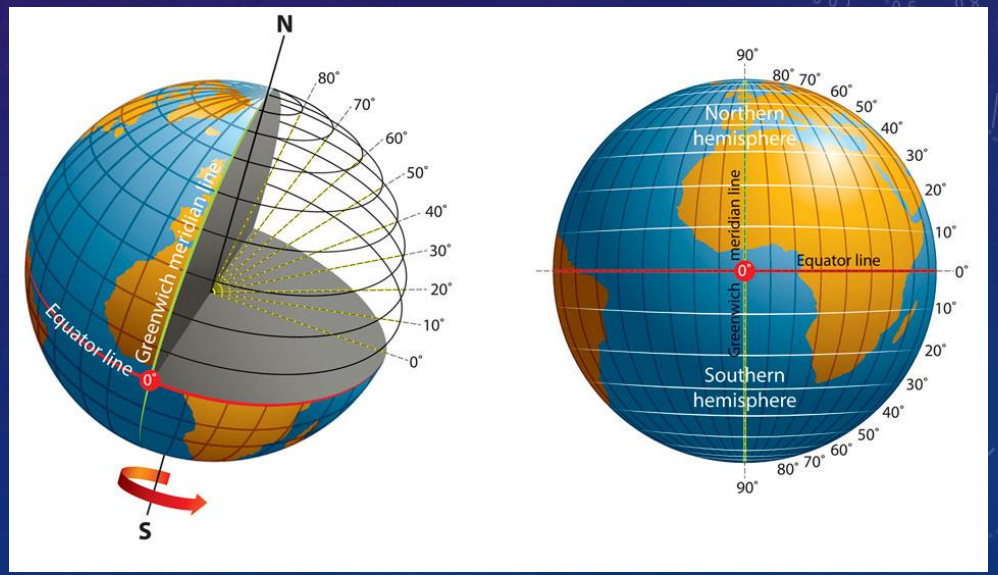
- Najlakše – posmatranje nestajanja i pojavljivanja satelita iza Jupitera
  - Satelit nestaje pre nego što stvarno „zađe“ iza Jupitera!
  - Pojavljivanje satelita kasni u odnosu na očekivano!
  - Jupiterova senka
- Astronomi su morali da uključe senku (položaj Sunca, Jupitera i Zemlje) u proračun efemerida
  - Sateliti – kazaljke časovnika



# GEOGRAFSKA ŠIRINA I DUŽINA

- U 17 veku nisu postojali tačni časovnici
  - Jupiterovi sateliti značajni za određivanje tačnog vremena
- Geografska širina – lako (zvezda Severnjača)
- Geografska dužina…? ☹
  - Britanska kraljevska opservatorija, Grinič (od 1884. god)
  - Mnogo moćna Britanska mornarica ☺
- Ne postoji mogućnost direktnog merenja
  - Greška od par minuta → nekoliko kilometara razlike
  - Prvi tačan časovnik za brodove, 1764 (Džon Harison)



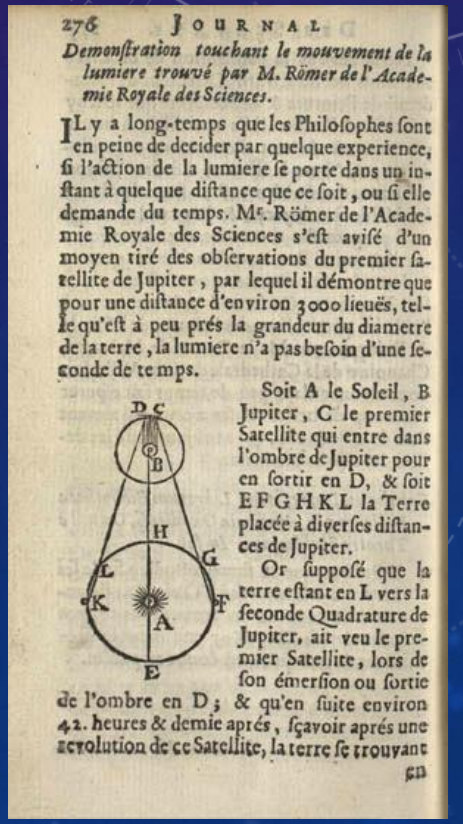



# GEOGRAFSKA DUŽINA

- Ako znamo razliku u vremenu između lokalnog vremena i „Griniča“
  - Za 24 h Zemlja napravi pun krug (360 stepeni), 15 stepeni/sat
  - Za 1 stepen potrebna su 4 minuta
- Brod krene iz Londona ka Atlantskom okeanu
  - Posle nekoliko nedelja posada vidi samo okean, nema kopna
  - Pred jutro, na osnovu Severnjače lako se odredi geografska širina (npr. $47^o$)
  - Npr. Sunce izlazi u 8:00 h (prema tačnom časovniku na ruci, koji pokazuje vreme u Londonu)
  - Kako je 22. mart dan prolećne ravnodnevice Sunce treba da izađe u 6:00 h ! ! !
  - Dva sata razlike → 30 stepeni geografske dužine
    - Ako je tačnost časovnika jedna sekunda – tačna pozicija sa greškom oko 2 km.

# PROBLEM?

- Galilej – otkrio satelite i konstruisao specijalan teleskop za njihovo precizno posmatranje na moru
  - Oko 150 godina pre Harisonovog preciznog časovnika za brodove
- Nažalost još nisu postojale precizne efemeride
- Francuski astronom – Đovani Kasini
  - Precizno merio rotaciju Jupitera (otkrio i pojaseve na Jupiteru)
- Remer posmatrao samostalno Jupiter, 1672. otišao u Paris kod Kasinija
- Kasini – merio vreme zalaska Io (beležio kad satelit izađe iz senke)
- Podaci pokazali – kad su Jupiter i Zemlja blizu efemeride su tačne
  - Kad su Jupiter i Zemlja daleko – pomračenje kasni 10 – 12 minuta u odnosu na predviđeno
  - Potrebno vreme da svetlost pređe razliku u rastojanju! (Kasini promenio mišljenje, smatrao da je brzina svetlosti beskonačna, problem je negde drugo…)
  - Remer – podržao prvo Kasinijevo tumačenje i izmerio brzinu svetlosti (70% tačne vrednosti)



276 JOURNAL

*Demonſtration touchant le mouvement de la lumiere trouvé par M. Römer de l'Academie Royale des Sciences.*

IL y a long-temps que les Philoſophes ſont en peine de decider par quelque experience, ſi l'action de la lumiere ſe porte dans un inſtant à quelque diſtance que ce ſoit, ou ſi elle demande du temps. Mr. Römer de l'Academie Royale des Sciences s'eſt aviſé d'un moyen tiré des obſervations du premier ſatellite de Jupiter, par lequel il démontre que pour une diſtance d'environ 3000 lieuës, telle qu'eſt à peu prés la grandeur du diametre de la terre, la lumiere n'a pas beſoin d'une ſeconde de temps.



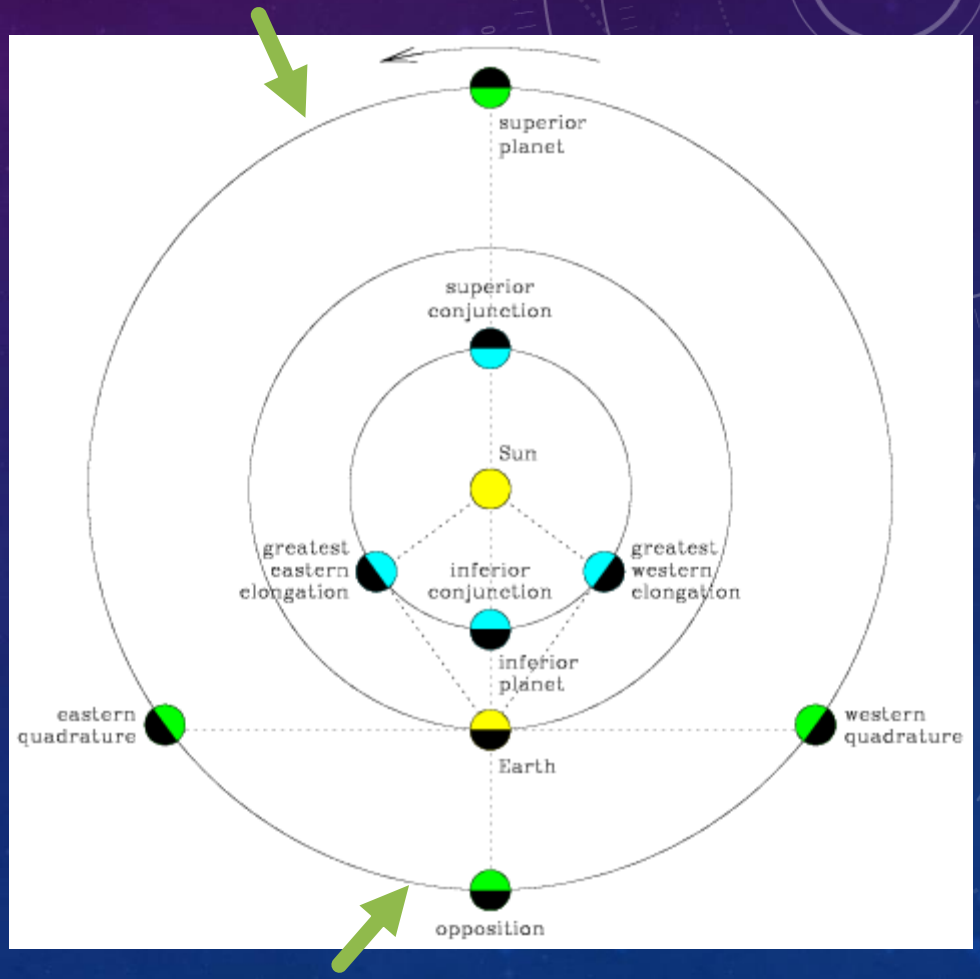

Soit A le Soleil, B Jupiter, C le premier Satellite qui entre dans l'ombre de Jupiter pour en ſortir en D, & ſoit E F G H K L la Terre placée à diverſes diſtances de Jupiter.

Or ſuppoſé que la terre eſtant en L vers la ſeconde Quadrature de Jupiter, ait veu le premier Satellite, lors de ſon émerſion ou ſortie de l'ombre en D; & qu'en ſuite environ 42. heures & demie aprés, ſçavoir aprés une revolution de ce Satellite, la terre ſe trouvant en

# IZBOR DATUMA

- Preporuka:
  - 2-3 meseca posle konjunkcije (1)
  - 1-2 meseca pre opozicije (2)

- Na primer:
  - konjunkcija - 29. januar 2021. godine
  - opozicija – 20. avgust 2021. godine

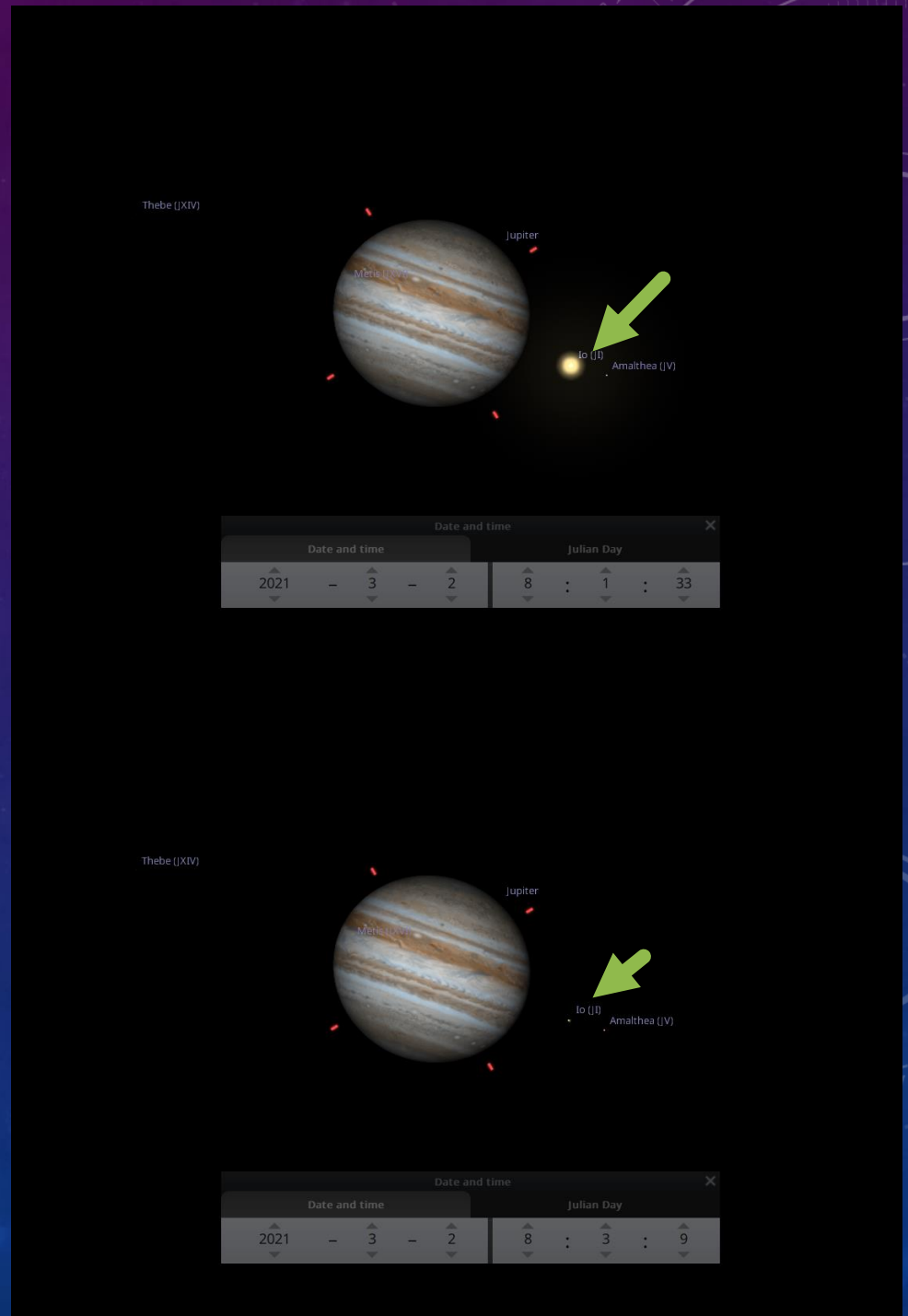


# PRVO POSMATRANJE

- **2. mart 2021. godine**
- Početak pomračenja Io
  - Vreme: 07:03:00,0
  - JD: 2459275,793750

- Procena za drugo posmatranje
  - npr. 15. jul 2021
  - Broj dana: 135
  - Period Io: 1,769861 dana
  - Broj orbita: $\frac{135}{1{,}769861} = 76{,}28 \approx 76$
    - uvek izabrati manji broj, tj. početi posmatranje ranije

konjunkcija - 29. januar 2021. godine
opozicija – 20. avgust 2021. godine



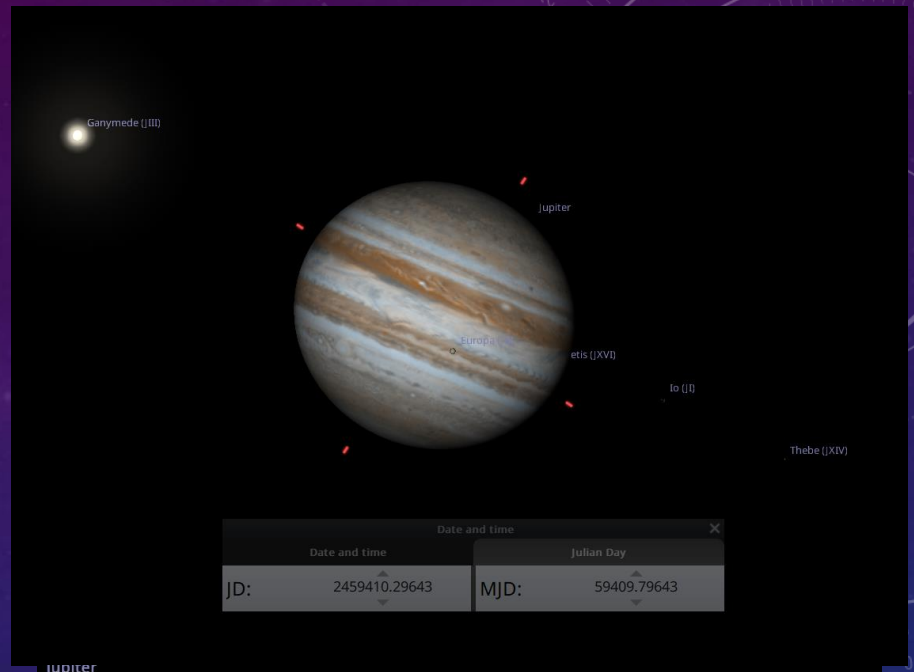
# DRUGO POSMATRANJE

- Očekivano vreme pomračenja(JD)
  - JD prvog posmatranja + broj orbita * period Io
  - JD (2) = 2459275,793750 + 76 * 1,769861
  - JD (2) = 2459410,303186

- Uneti približno vreme **pre** očekivanog pomračenja!

# DRUGO POSMATRANJE - REZULTATI

- **14. jul 2021**
  - Vreme: 19:06:00,0
  - JD 2459410,295833
- Izračunati razliku u vremenu predviđenog i posmatranog vremena
  - 2459410,303186 – 2459410,295833
  - $\Delta$JD = 0,007353 dana = 635,2992 sekundi
- Razlika u rastojanju
  - D = 5,95649 AU – 4,20139 AU
  - D = 1,7551 AU




# BRZINA SVETLOSTI

- $\Delta T$ = 635,2992 sekundi
- D = 1,7551 AU = 262559223,4 km

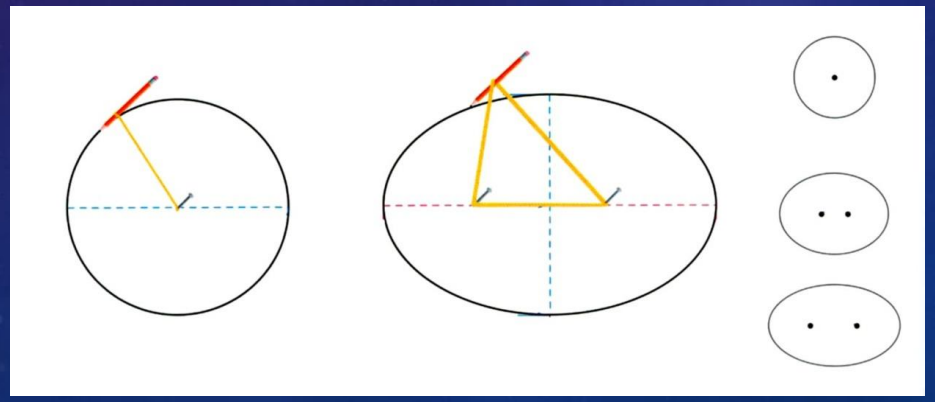
- Brzina svetlosti c = D / $\Delta T$
- c = 413.284,4 km/s

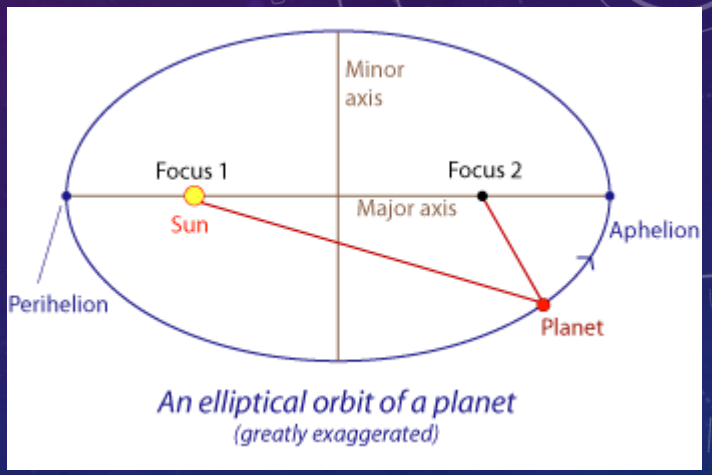
- Oko 35% veća vrednost od tačne

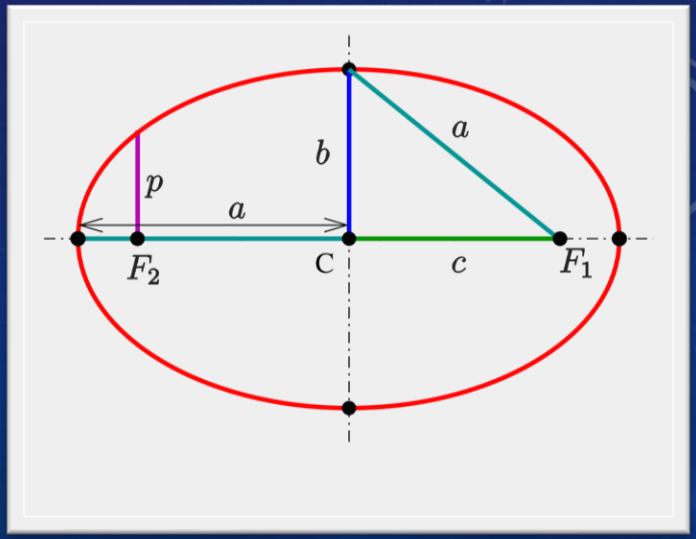
# KEPLEROVI ZAKONI

# KEPLEROVI ZAKONI (1. ZAKON)

- ***Planete se oko Sunca kreću po eliptičkim putanjama, u čijoj se zajedničkoj žiži nalazi Sunce***
- Jedačina elipse $r = \frac{p}{1+\varepsilon \cos\theta}$
  - Perihel $\theta = 0^{o}$, afel $\theta = 180^{o}$
  - Za $\theta = 90^{o}$ i $\theta = 270^{o}$ – rastojanje $p$

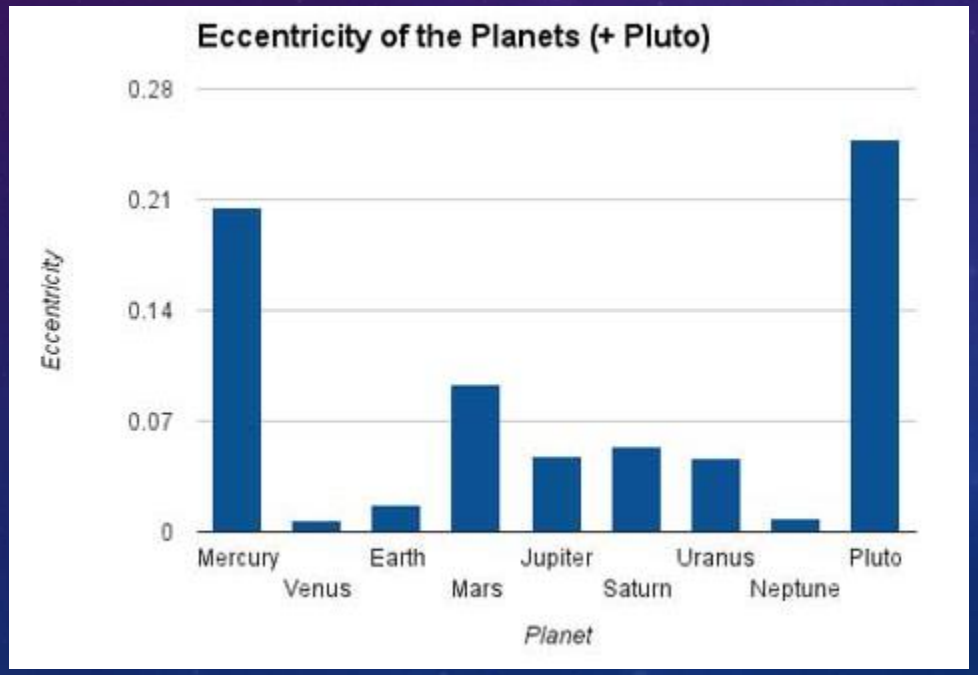


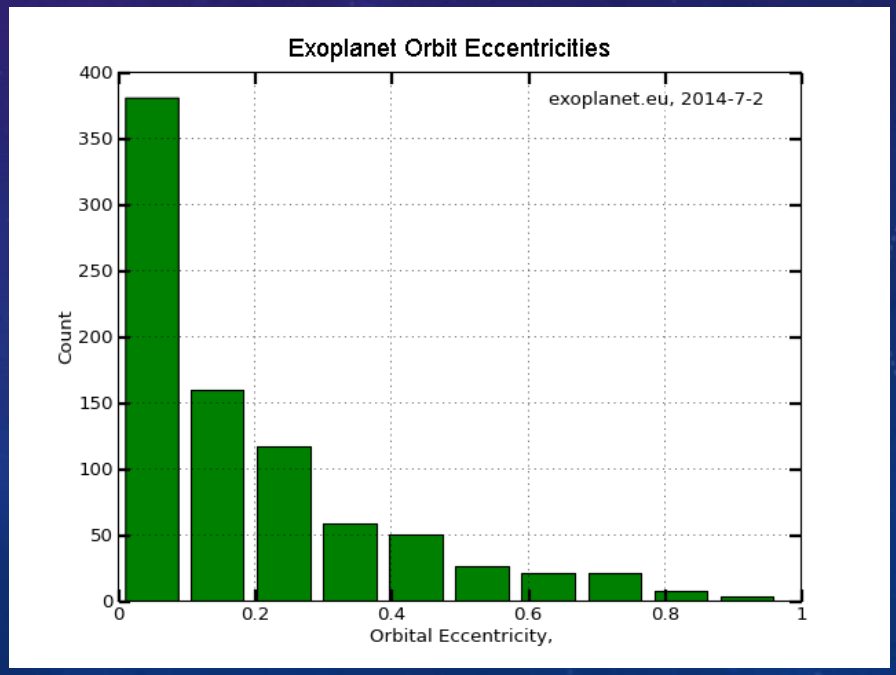


# KEPLEROVI ZAKONI (1. ZAKON)

- Putanje planeta su malog ekscentriciteta (ne razlikuju se mnogo od kružnih), osim u  slučaju Merkura i “bivše” planete Pluton.
- Eliptične putanje imaju i sateliti planeta, asteroidi i periodične komete.



# KEPLEROVI ZAKONI (2. ZAKON)

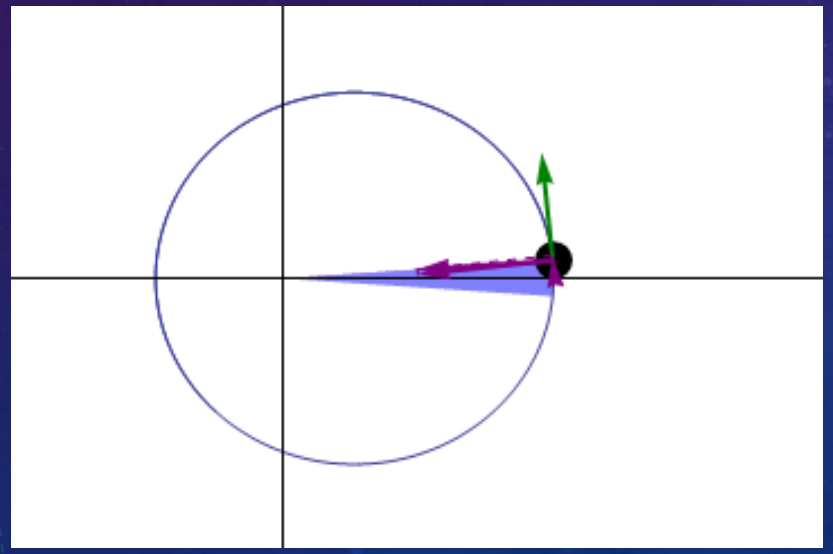
$$dA = \frac{1}{2} r \cdot r d\theta$$

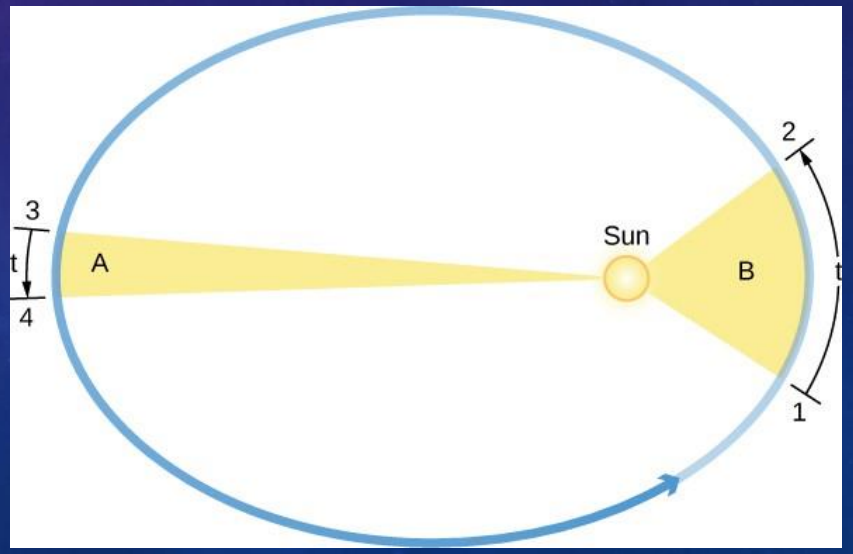
$$\frac{dA}{dt} = \frac{r^2}{2} \frac{d\theta}{dt}$$

$$P \cdot \frac{r^2}{2} \frac{d\theta}{dt} = \pi ab$$

$$n = \frac{2\pi}{P} \quad \Rightarrow \quad r^2 d\theta = abndt$$

- **Radijus vektor planete u jednakim vremenskim intervalima opisuje jednake površine**
- Zbog ovoga se planeta kreće brže u blizini perihela, a sporije u blizini afela.





# KEPLEROVI ZAKONI (3. ZAKON)

- **Kvadrati perioda (*T*) obilaska planete oko Sunca srazmerni su kubovima velikih poluosa (*a*) njihovih putanja**

$$\frac{a_1^3}{T_1^2} = \frac{a_2^3}{T_2^2} = \frac{a_3^3}{T_3^2} = \cdots const$$

- omogućuje da se na osnovu poznavanja perioda obilaska Zemlje oko Sunca i velike poluose njene putanje odrede elementi kretanja drugih planeta.

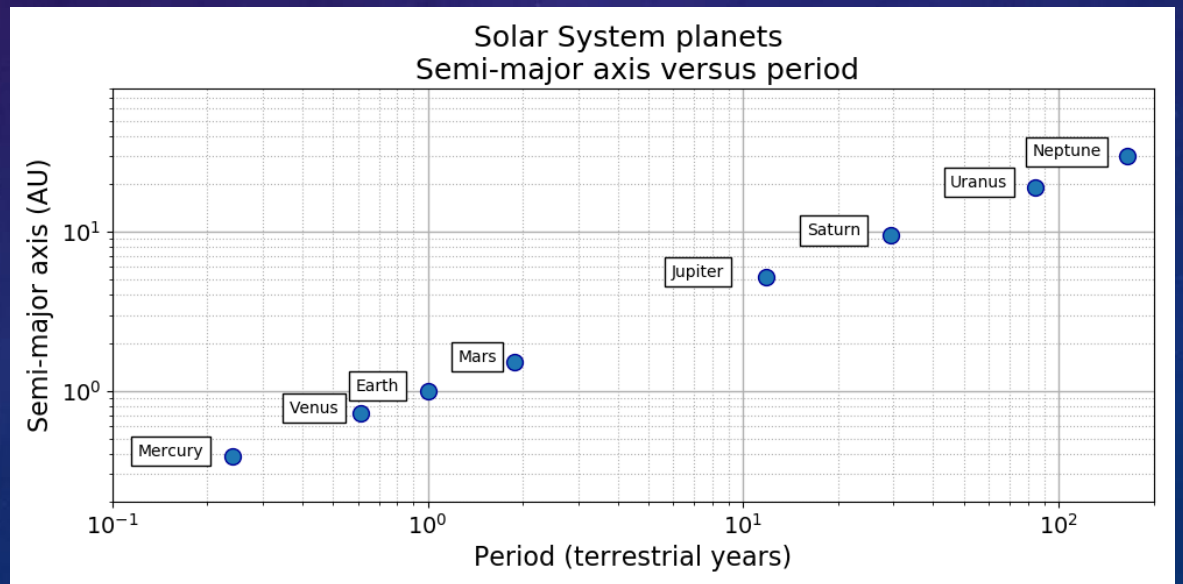
# KEPLEROVI ZAKONI (3. ZAKON)

**Data used by Kepler (1618)**

| Planet | Mean distance to sun (AU) | Period (days) | $\frac{R^3}{T^2}$ ($10^{-6}$ $AU^3/day^2$) |
|---|---|---|---|
| Mercury | 0.389 | 87.77 | 7.64 |
| Venus | 0.724 | 224.70 | 7.52 |
| Earth | 1 | 365.25 | 7.50 |
| Mars | 1.524 | 686.95 | 7.50 |
| Jupiter | 5.2 | 4332.62 | 7.49 |
| Saturn | 9.510 | 10759.2 | 7.43 |

**Modern data (Wolfram Alpha Knowledgebase 2018)**

| Planet | Semi-major axis (AU) | Period (days) | $\frac{R^3}{T^2}$ ($10^{-6}$ $AU^3/day^2$) |
|---|---|---|---|
| Mercury | 0.38710 | 87.9693 | 7.496 |
| Venus | 0.72333 | 224.7008 | 7.496 |
| Earth | 1 | 365.2564 | 7.496 |
| Mars | 1.52366 | 686.9796 | 7.495 |
| Jupiter | 5.20336 | 4332.8201 | 7.504 |
| Saturn | 9.53707 | 10775.599 | 7.498 |
| Uranus | 19.1913 | 30687.153 | 7.506 |
| Neptune | 30.0690 | 60190.03 | 7.504 |



# TREĆI KEPLEROV ZAKON

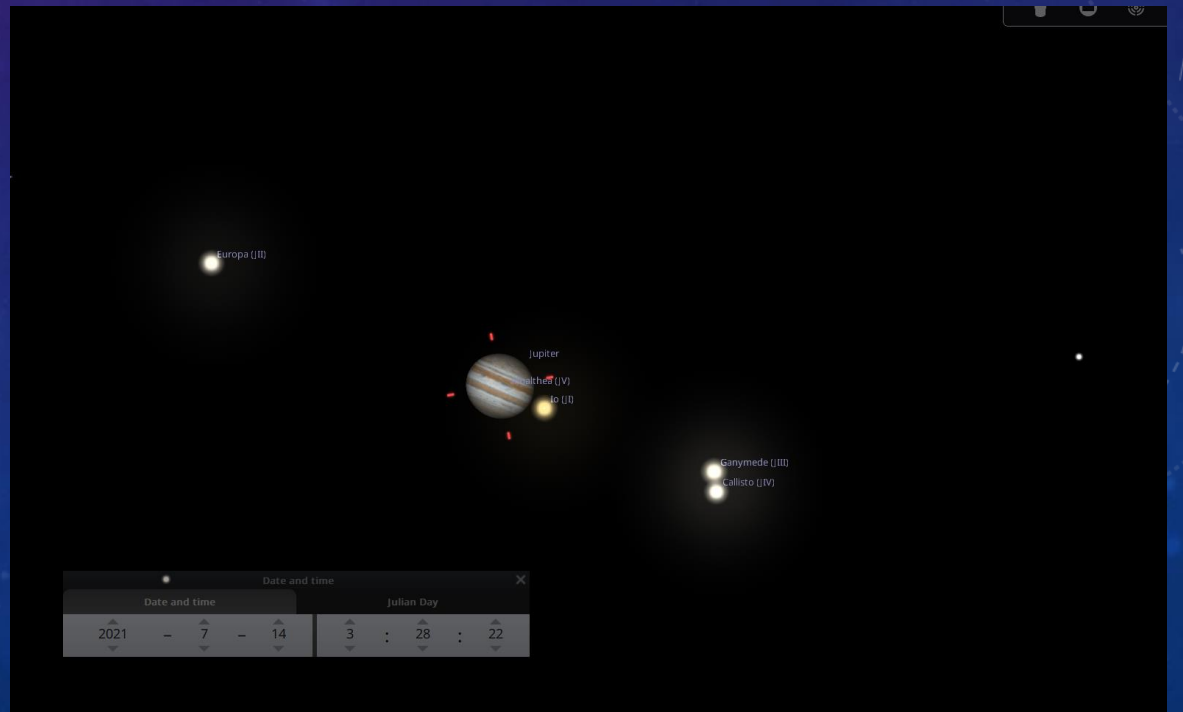
- Posmatranje Galilejevih satelita

$$\frac{T^2}{a^3} = \frac{4\pi^2}{G \cdot M}$$

$$M = \frac{a^3}{T^2}$$

- $M$ – masa tela (u solarnim masama),
- $a$ – poluosa (u astronomskim jedinicama),
- $T$ – period (u godinama)

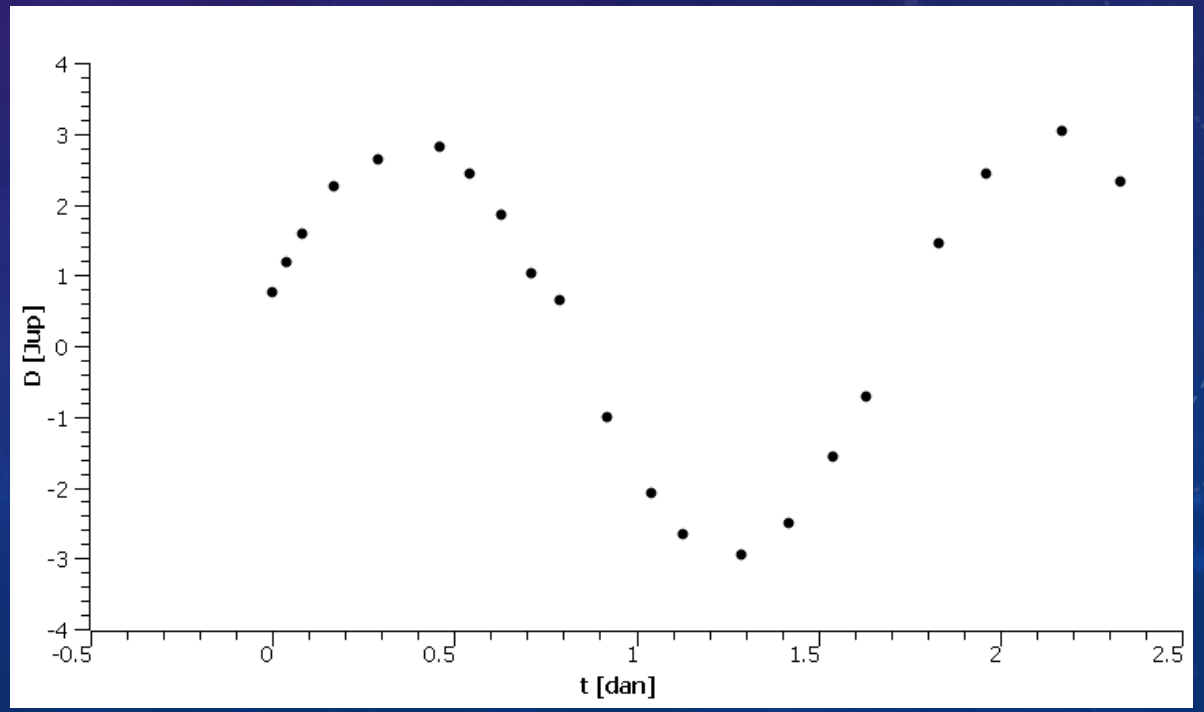
$$\frac{T^2}{a^3} = \frac{4\pi^2}{G \cdot (m_1 + m_2)}$$

# MERENJE

| vreme | T | sig | Min | sec | Jupiter | D [Jup] | D [AU] |
|---|---|---|---|---|---|---|---|
| 2459409.565 | 0.00 | 1 | 0 | 35.41 | 46.51 | 0.7613 | 1.63E-05 |
| 2459409.607 | 0.04 | 1 | 0 | 55.26 | 46.51 | 1.1881 | 2.55E-05 |
| 2459409.649 | 0.08 | 1 | 1 | 13.87 | 46.51 | 1.5883 | 3.41E-05 |
| 2459409.732 | 0.17 | 1 | 1 | 44.86 | 46.51 | 2.2546 | 4.84E-05 |
| 2459409.857 | 0.29 | 1 | 2 | 2.12 | 46.51 | 2.6257 | 5.63E-05 |
| 2459410.024 | 0.46 | 1 | 2 | 11.16 | 46.51 | 2.8200 | 6.05E-05 |
| 2459410.107 | 0.54 | 1 | 1 | 53.27 | 46.51 | 2.4354 | 5.23E-05 |
| 2459410.191 | 0.63 | 1 | 1 | 26.67 | 46.51 | 1.8635 | 4.00E-05 |
| 2459410.274 | 0.71 | 1 | 0 | 47.87 | 46.51 | 1.0292 | 2.21E-05 |
| 2459410.357 | 0.79 | 1 | 0 | 30.00 | 46.51 | 0.6450 | 1.38E-05 |
| 2459410.482 | 0.92 | -1 | 0 | 47.08 | 46.51 | -1.0123 | -2.17E-05 |
| 2459410.607 | 1.04 | -1 | 1 | 36.94 | 46.51 | -2.0843 | -4.47E-05 |
| 2459410.691 | 1.13 | -1 | 2 | 3.28 | 46.51 | -2.6506 | -5.69E-05 |
| 2459410.857 | 1.29 | -1 | 2 | 16.76 | 46.51 | -2.9404 | -6.31E-05 |
| 2459410.982 | 1.42 | -1 | 1 | 56.61 | 46.51 | -2.5072 | -5.38E-05 |
| 2459411.107 | 1.54 | -1 | 1 | 12.66 | 46.51 | -1.5622 | -3.35E-05 |
| 2459411.191 | 1.63 | -1 | 0 | 33.35 | 46.51 | -0.7171 | -1.54E-05 |
| 2459411.399 | 1.83 | 1 | 1 | 7.69 | 46.51 | 1.4554 | 3.12E-05 |
| 2459411.524 | 1.96 | 1 | 1 | 53.30 | 46.51 | 2.4360 | 5.23E-05 |
| 2459411.732 | 2.17 | 1 | 2 | 21.25 | 46.51 | 3.0370 | 6.52E-05 |
| 2459411.899 | 2.33 | 1 | 1 | 48.32 | 46.51 | 2.3290 | 5.00E-05 |

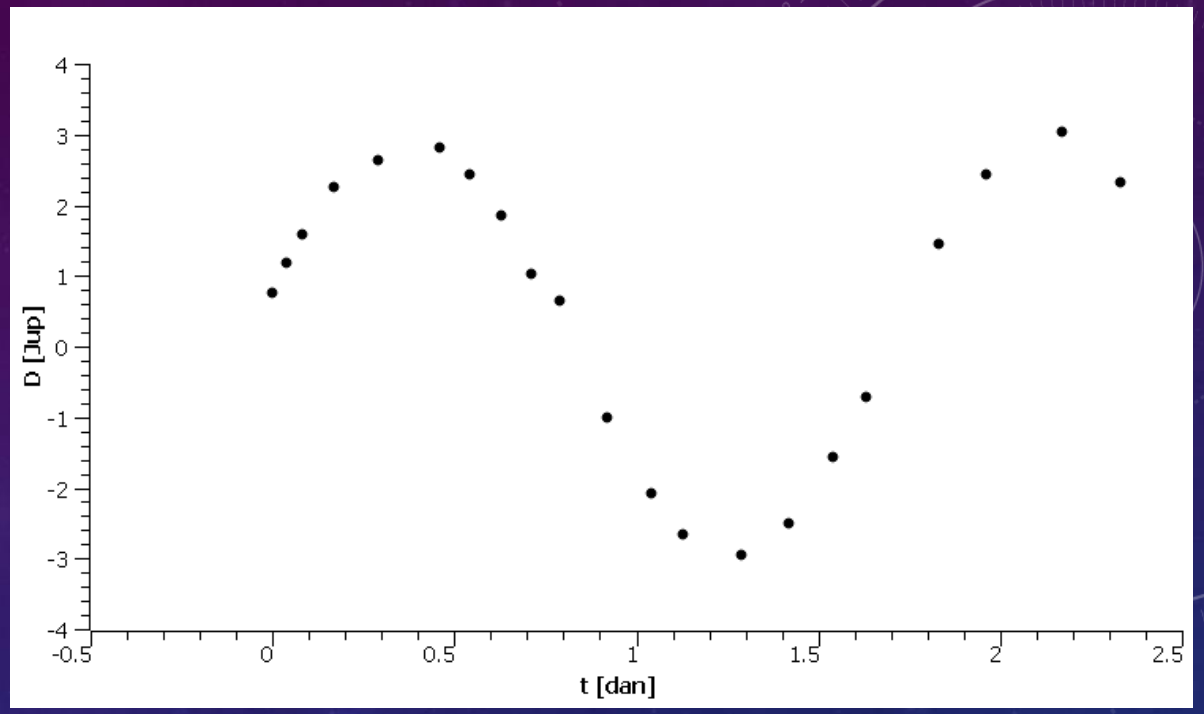
| | |
|---|---|
| **Jupiter [km]** | 139820 |
| **AU [km]** | 149597871 |
| **Jupiter [AU]** | 46606.67 |

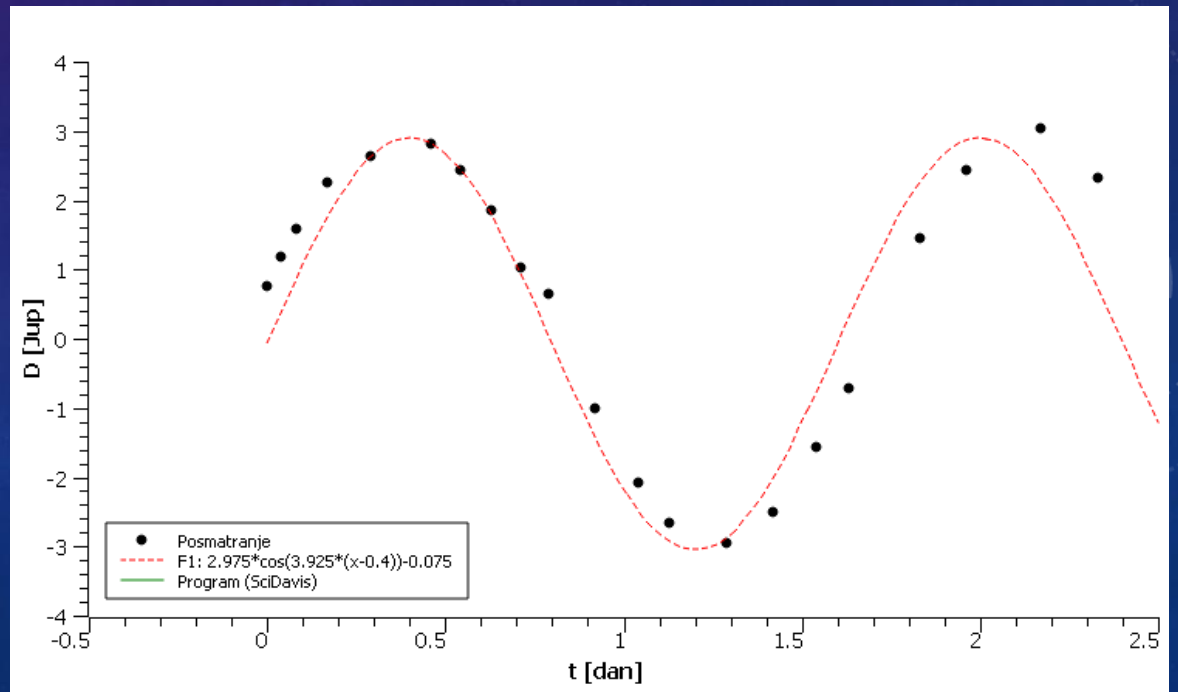


# MERENJE

$$y = a\cos\big(\omega\,(t-c)\big) + b$$

- $b = \frac{1}{2}\big(2.9 + (-3.05)\big) = \frac{-0.15}{2} = -0.075$
- $a = \frac{1}{2}\big(2.9 - (-3.05)\big) = \frac{5.95}{2} = 2.975$
- $T = \frac{2\pi}{\omega} = 2(2.1 - 1.3) = 2 \cdot 0.8 = 1.6$
  - $\omega = \frac{2\pi}{T} = 3.925$
- $c = 0.4$



# „MALO“ TAČNIJE

Function: user1 (x, a, b, c, w)

a*cos(w*(x-c))+b

Initial guesses

| Parameter | Value | Constant |
|---|---|---|
| a | -2.955786850733 | ☐ |
| b | 0.0408699848125348 | ☐ |
| c | 1.25869498609216 | ☐ |
| w | -3.55636264712317 | ☐ |

Algorithm: Scaled Levenberg-Marquardt

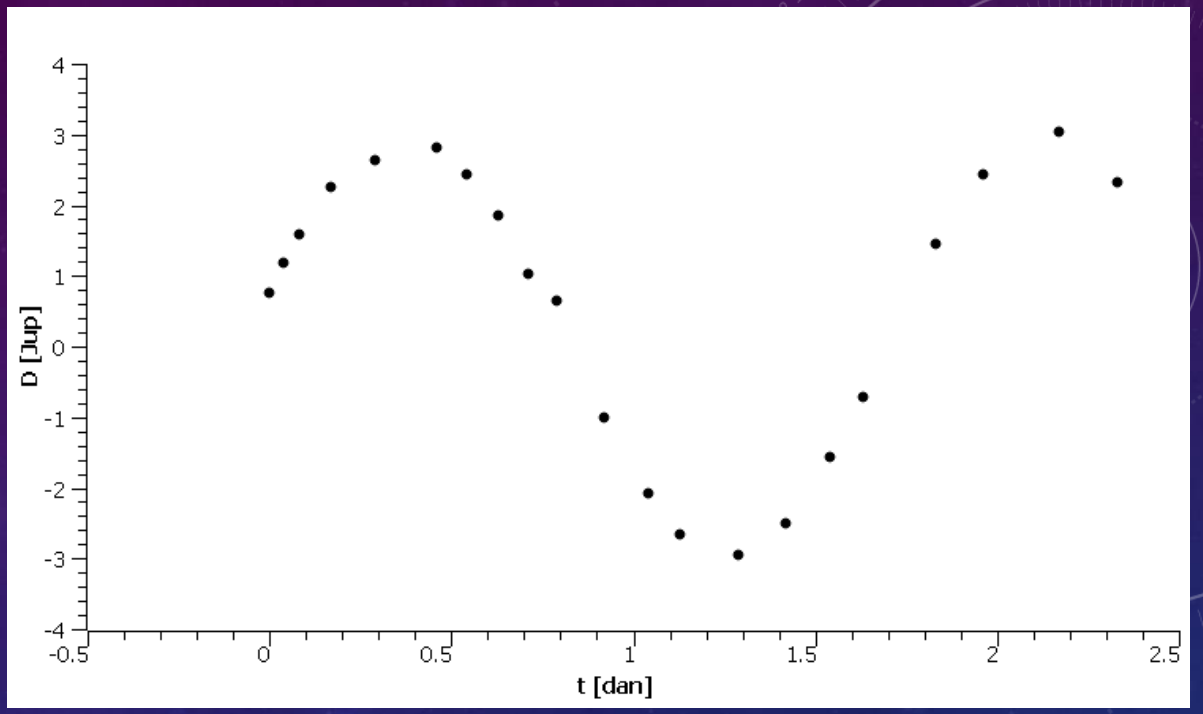
Color

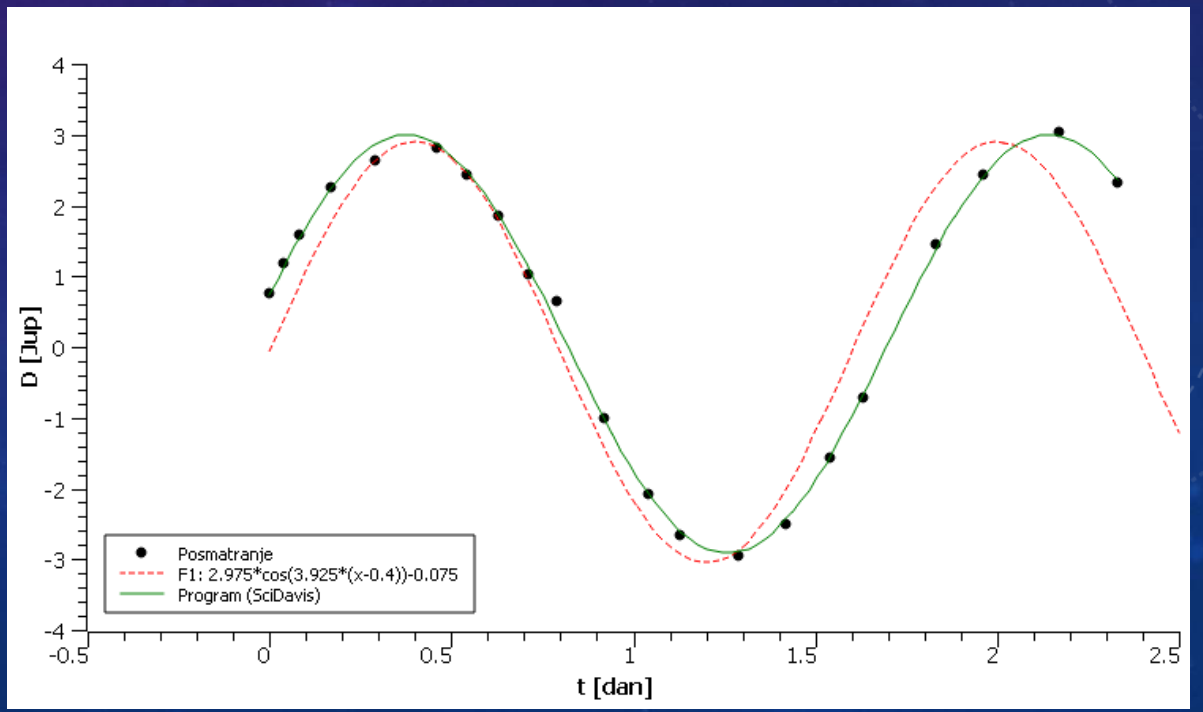
From x= 0

To x= 2.33

Iterations 1000

Tolerance 1e-4




$b = -0.075, a = 2.975, \omega = 3.925, c = 0.4$

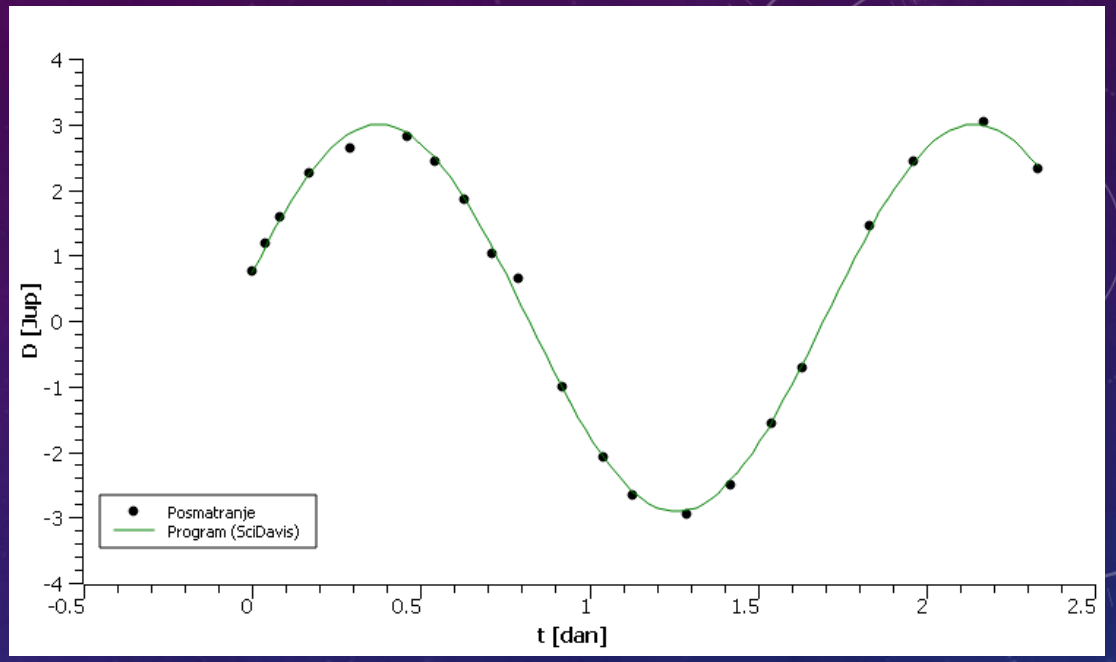
$$M = \frac{a^3}{T^2}$$

# DA UPOREDIMO…

- „Papir i olovka“
  - $a = 2.956$ [Jup] $= 0.00276$ AU
  - $T = 1.6$ dana $= 0.0044$ god
  - $M = 0.0009134\ M_\odot$
- „Fitovanje“ podataka
  - $a = 2.975$ [Jup] $= 0.00278$ AU
  - $T = 1.766$ dana $= 0.0048$ god
  - $M = 0.0009311\ M_\odot$

$\Delta = 4.29\ \%$

$\Delta = 2.43\ \%$



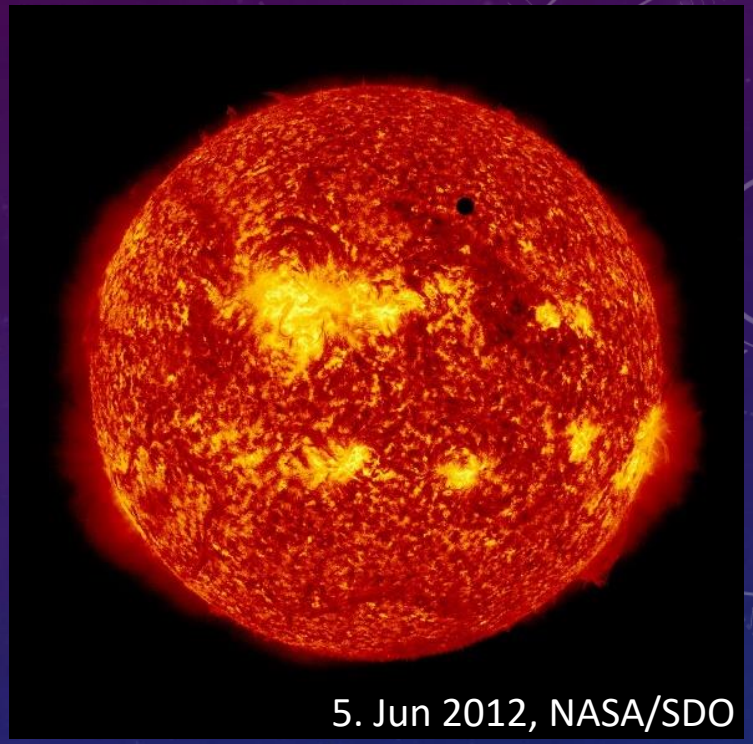

$M = 0.0009543\ M_\odot$

# TRANZIT MERKURA I VENERE

ODREĐIVANJE ASTRONOMSKE JEDINICE

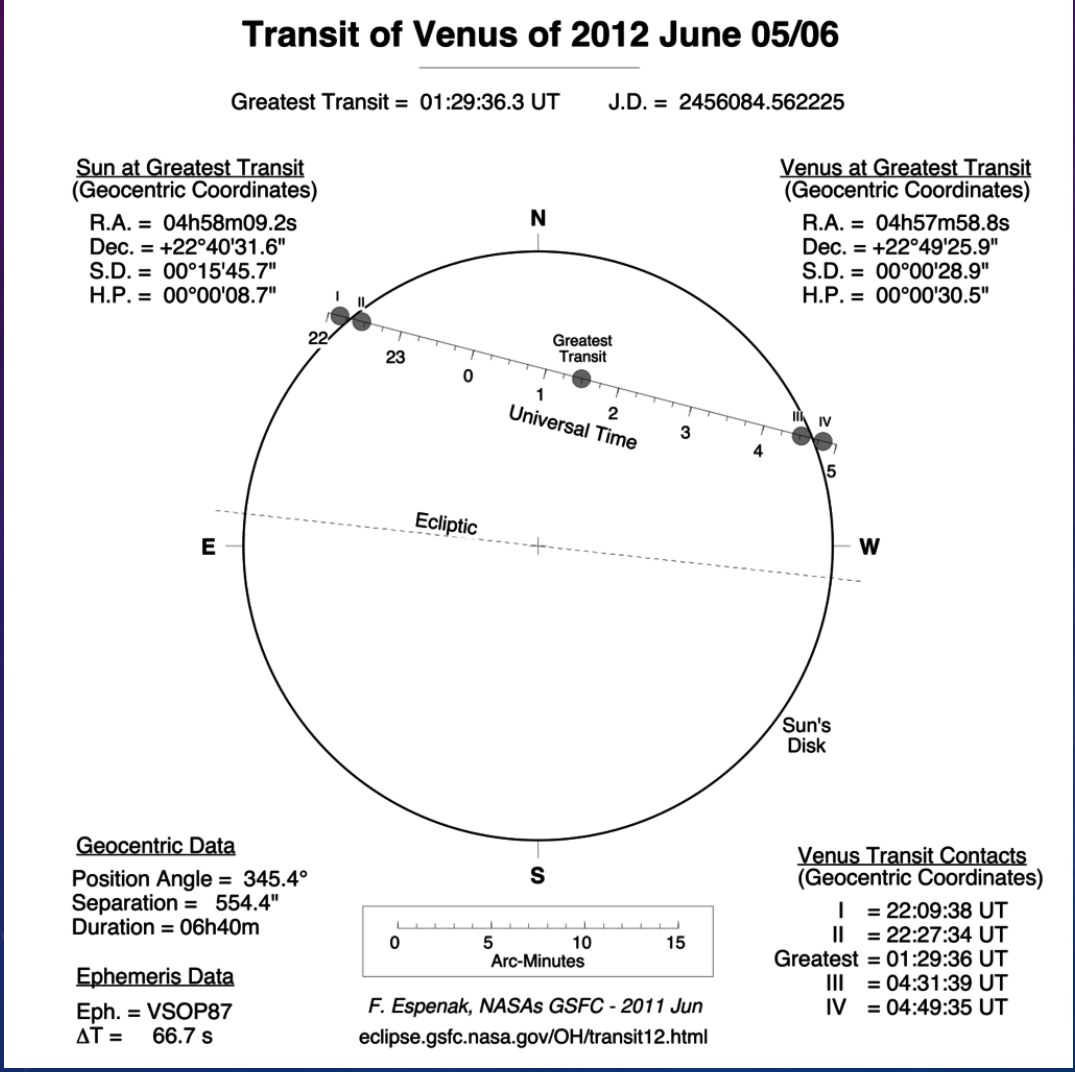
# TRANZITI MERKURA I VENERE

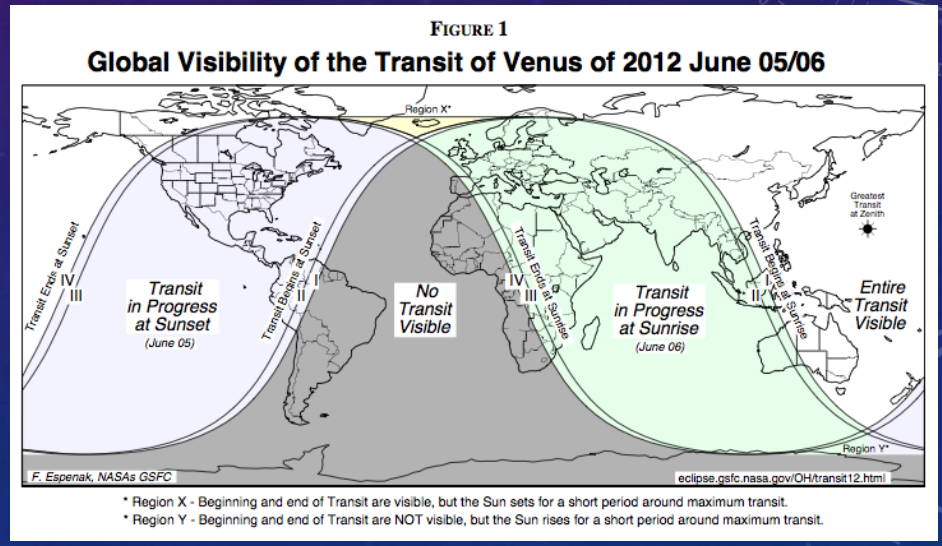
- Merkur – 1-2 puta u deceniji
- Venera – manje od dva puta po veku
  - Period 243 godine
  - Približno 105,8 – 8 – 121,5 – 8 godina
- Prvi zabeleženi tranzit Venere
  - 1639. godine, od tada 5
  - Poslednja dva 2004 i 2012
    - Sledeći 2117. odine ☹
- Merkur:
  - 2003, 2006, 2016 i 11. novembar 2019
  - Sledeći 2032. godine :/
- **3. jun 2014**
  - Rover Curiosity posmatrao tranzit sa Marsa
  - Prvi tranzit sa druge planete



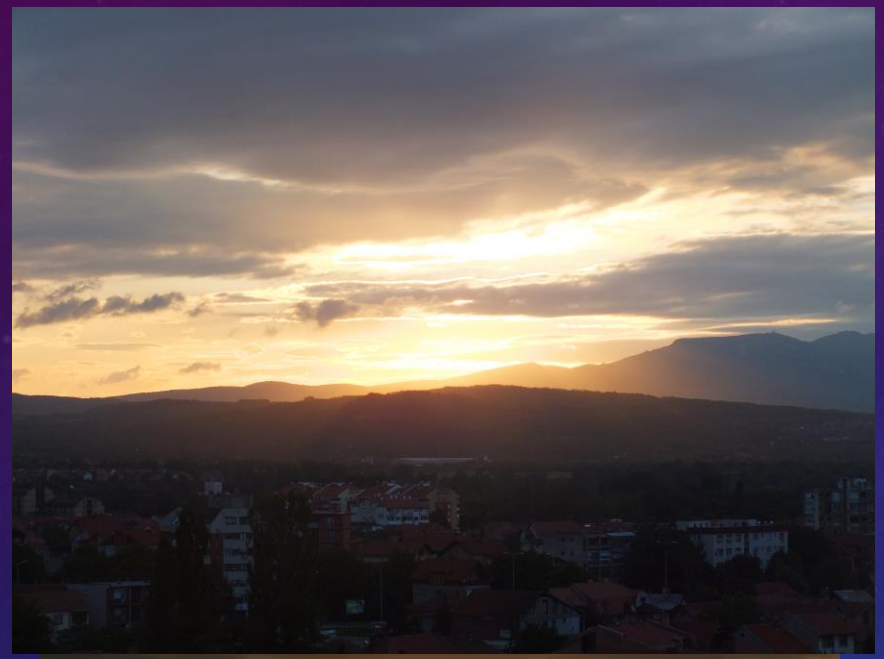
5. Jun 2012, NASA/SDO

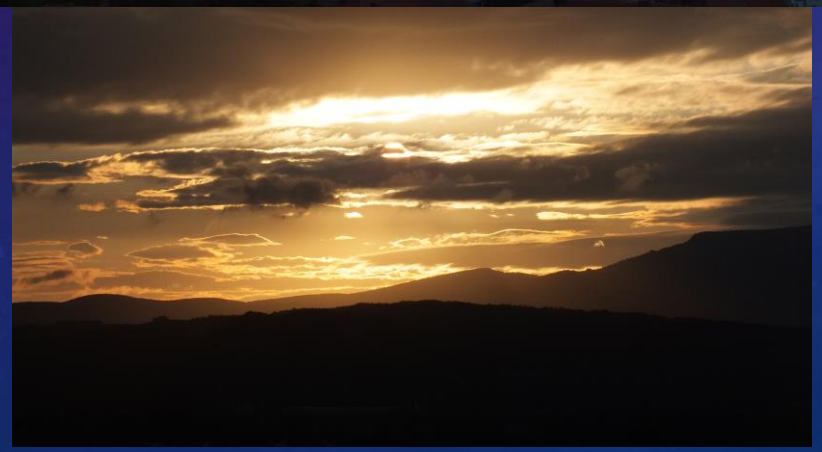
# TRANZIT VENERE, 2012. GODINA



# TRANZIT U NIŠU





# TRANZIT U NIŠU

N-E 06.06.2012 oko 5:45 h S-W

# TRANZITI VENERE I AU

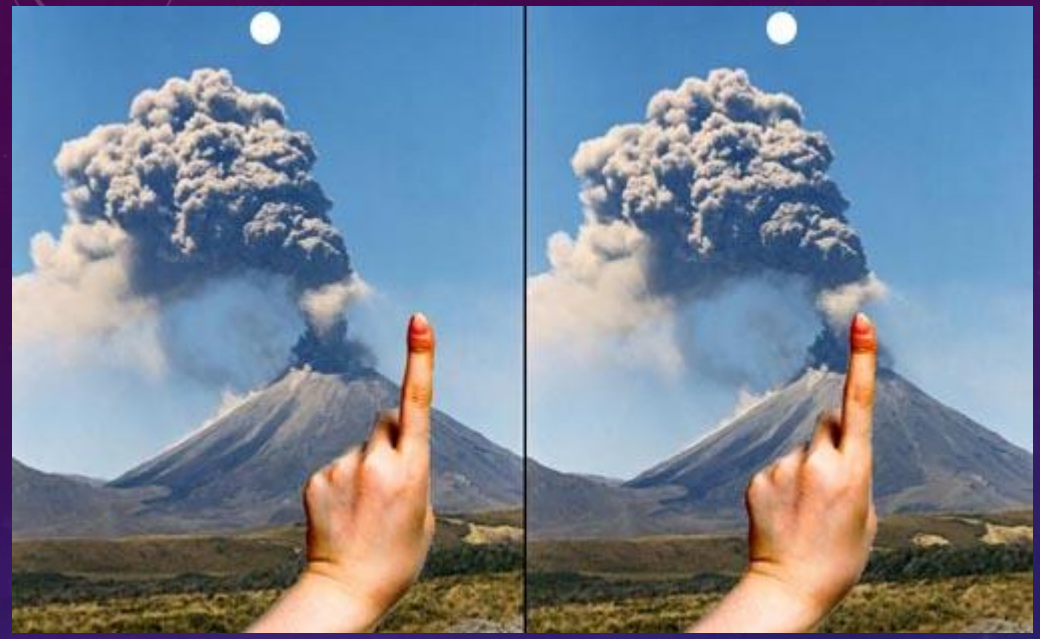
- Retki ali igrali važnu ulogu u istoriji astronomije
- 1761, 1769, 1874 i 1882. godina
- Velike ekspedicije u udaljene delove sveta
- Glavni razlog – merenje astronomske jedinice (AU)
- 1600 godina – radovi T. Brahe, J. Kepler relativna rastojanja
  - Merkur (0,39), Venera (0,72), Zemlja (1,00), Mars (1,52), Jupiter (5,20), Saturn (9,54)

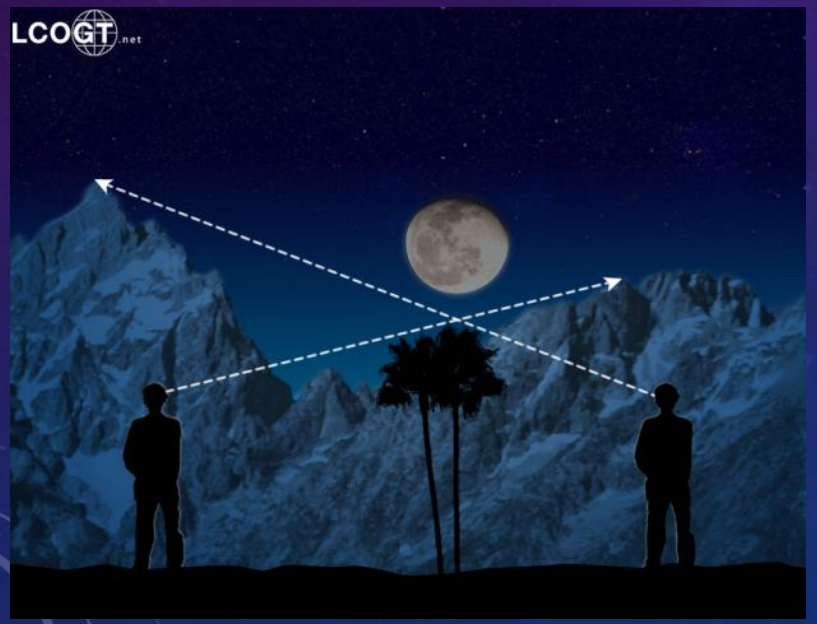
# TRANZITI VENERE I AU

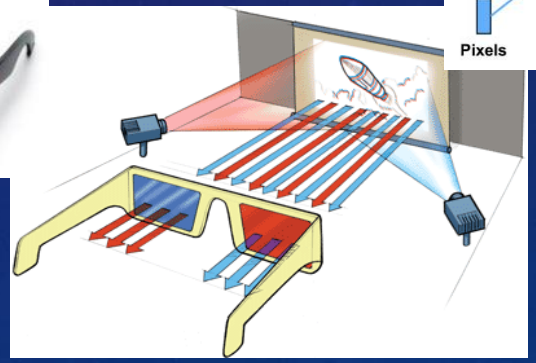
- Edmund Halej (1715)
  - Predložio metod za merenje AU
- Posmatranje tranzita Venere
- Dva posmatrača na dve različite lokacije
  - Istovremeno posmatraju
  - Vide „pomeraj“ (paralaksu) na površini Sunca
  - Meri se paralaksa, poznato rastojanje posmatrača
  - Trigonometrija → rastojanje do Venere

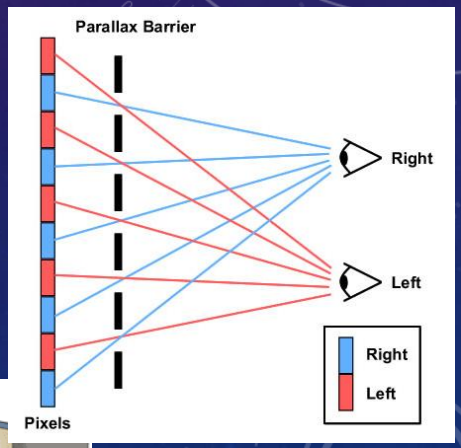
# PARALAKSA

- Na Zemlji, svakog dana



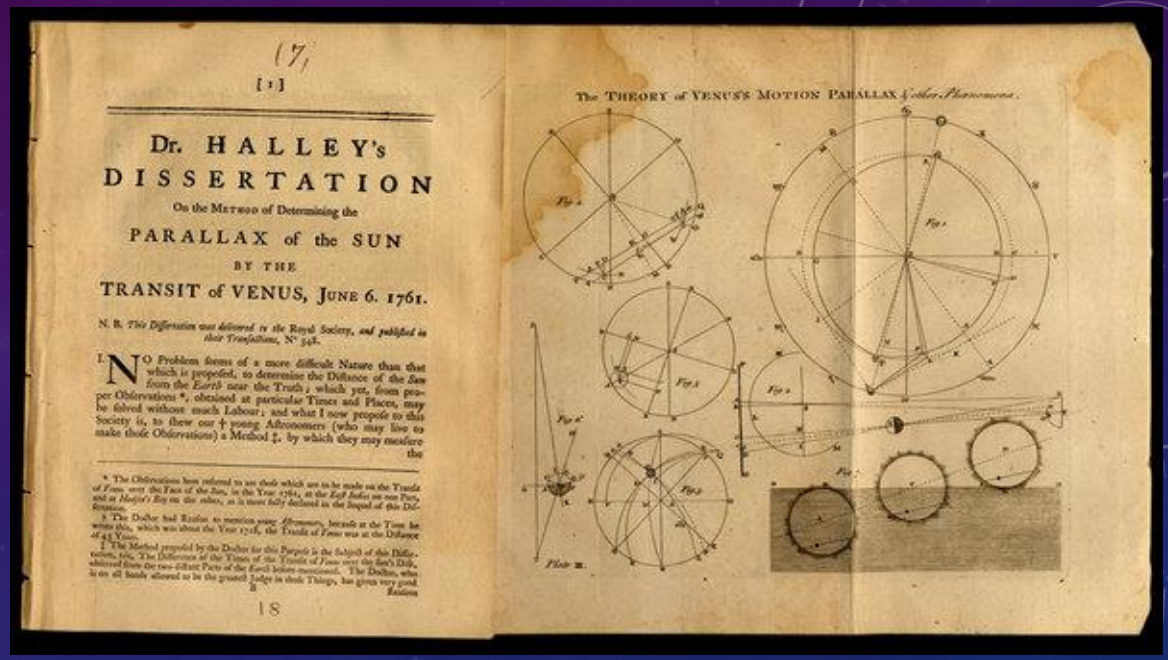





http://lcogt.net/spacebook/parallax-and-distance-measurement
www.parallaxperspective.com/2012/04/16/what-is-a-parallax-anyway/

# TRANZITI VENERE I AU



[1]

Dr. HALLEY's
DISSERTATION
On the METHOD of Determining the
PARALLAX of the SUN
BY THE
TRANSIT of VENUS, JUNE 6. 1761.

N. B. *This Dissertation was delivered to the Royal Society, and published in their Transactions, N° 348.*

NO Problem seems of a more difficult Nature than that which is proposed, to determine the Distance of the *Sun* from the *Earth* near the Truth; which yet, from proper Observations *, obtained at particular Times and Places, may be solved without much Labour: and what I now propose to this Society is, to shew our + young Astronomers (who may live to make those Observations) a Method ‡, by which they may measure the

* The Observations here referred to are those which are to be made on the Transit of *Venus* over the Face of the *Sun*, in the Year 1761, *at the East Indies* or one Part, and at *Hudson's Bay* on the other, as is more fully declared in the Sequel of this Dissertation.

+ The Doctor had Reason to mention *young Astronomers*, because at the Time he wrote this, which was about the Year 1718, the Transit of *Venus* was at the Distance of 43 Years.

‡ The Method proposed by the Doctor for this Purpose is the Subject of this Dissertation, viz. The Difference of the Times of the Transit of *Venus* over the *Sun's* Disk, observed from the two distant Parts of the *Earth* before-mentioned. The Doctor, who is on all hands allowed to be the greatest Judge in these Things, has given very good Reasons

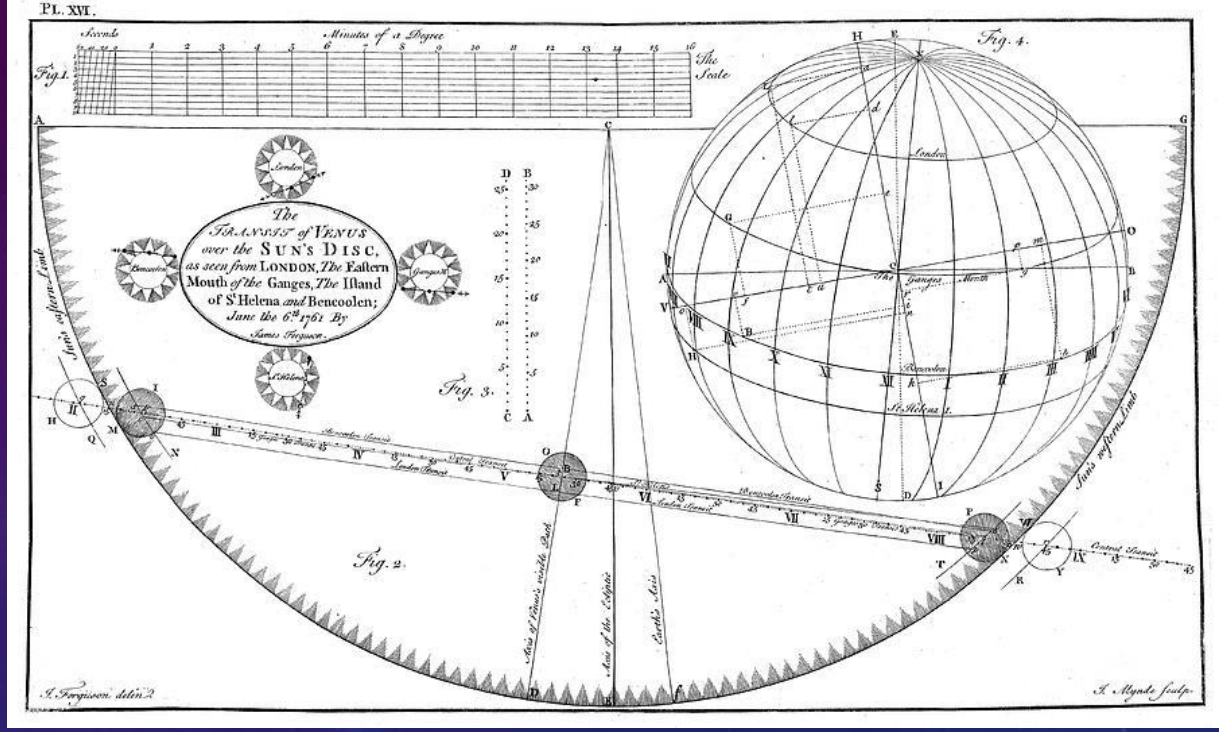
18

The THEORY of VENUS's MOTION PARALLAX & other Phænomena.

Plate II.

- Samo princip
  - Halej je znao da je proces kompleksniji
  - Za primenu paralakse – neophodno da posmatranje bude istovremeno
  - Problem: sinhronizacija časovnika na dugim putovanjima
- Halej zaključio da od pozicije posmatrača ne zavisi samo putanja Venere već i trajanje tranzita (1. i 4. kontakt

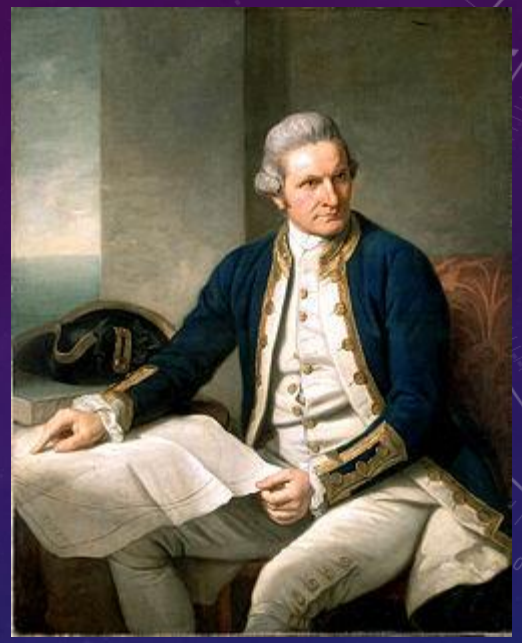
# TRANZITI VENERE I AU

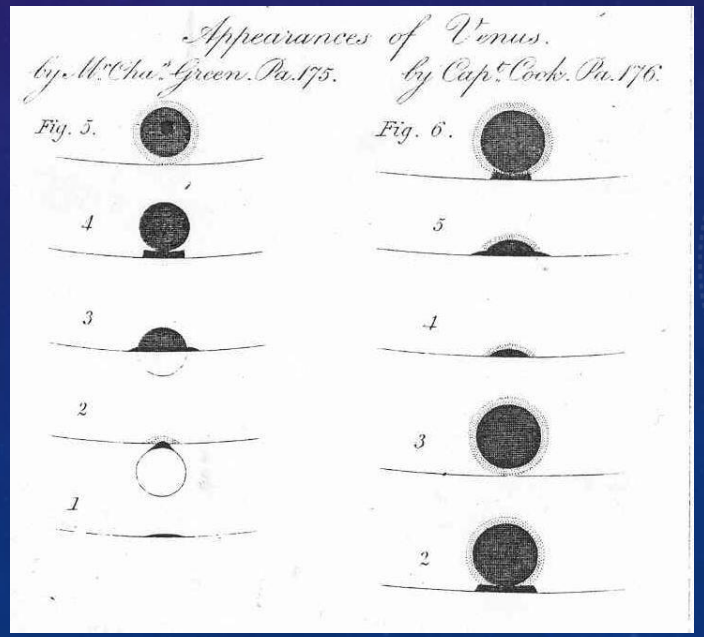



- E. Halej (1656-1742) nije video tranzit
- Primena metoda planirana za 1761. i 1769. godinu
  - Predviđanja za 1761. godinu
    - London, Indija (ušće reke Gang) i Malezija (Bencoolen)

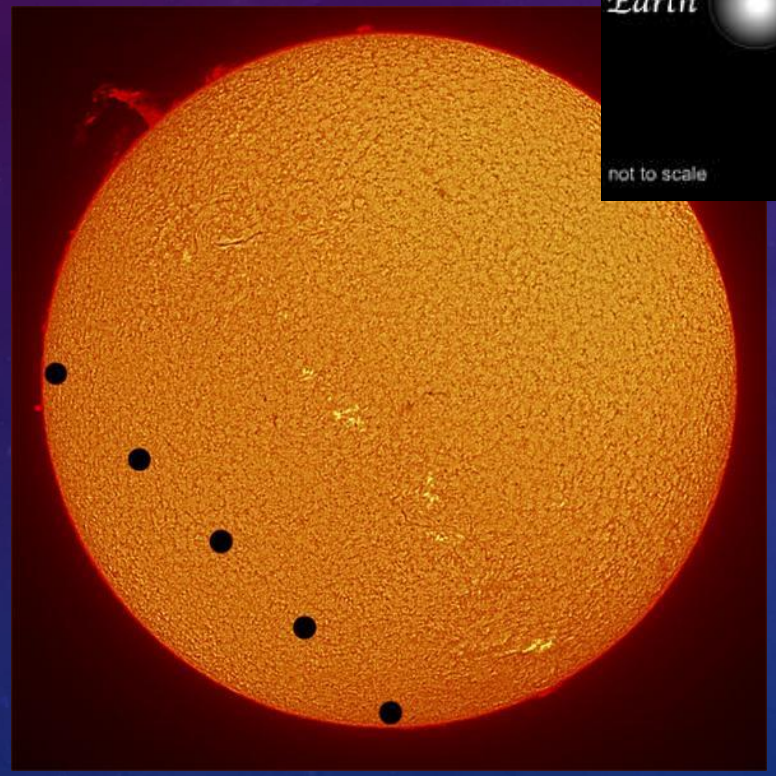
# TRANZITI VENERE I AU

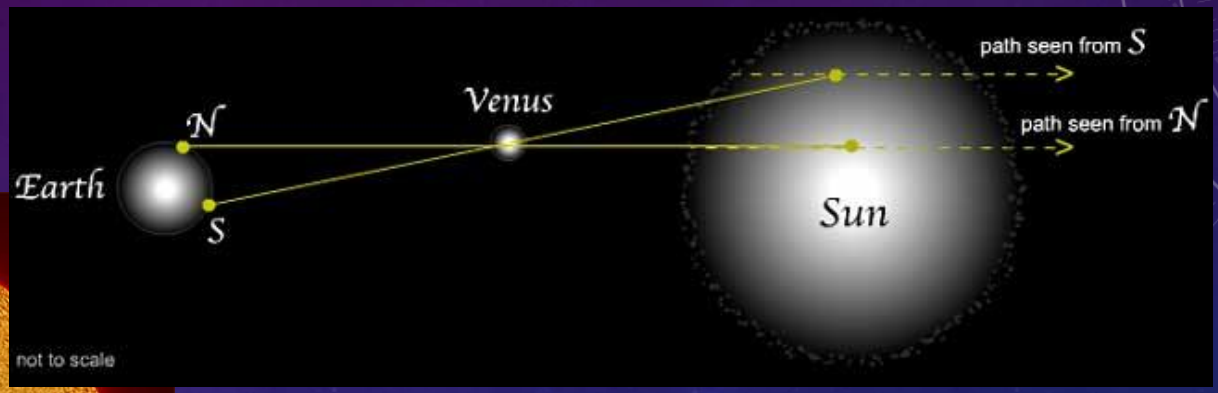
- Britanija i Francuska – velike ekspedicije
  - Kapetan Kuk – prvo putovanje u Južni Pacifik zbog tranzita 1769 (Tahiti)
- Većina imala dobre vremenske uslove
  - Rezultati razočarali ☹
  - Najveći problem određivanje vremena (1. kontakt)
- Sledeći tranziti 1874 i 1882
  - Napredovala tehnologija, ali loši rezultati
- Sledeći tranzit XX vek
  - 2004. godina (CLEA vežba)

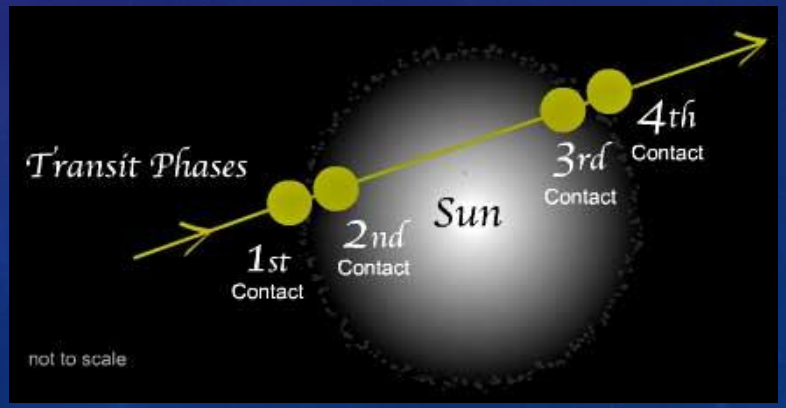




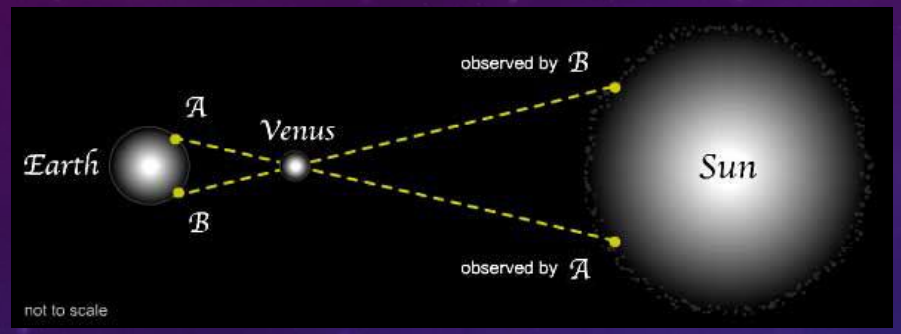
# RASTOJANJE ZEMLJA - SUNCE

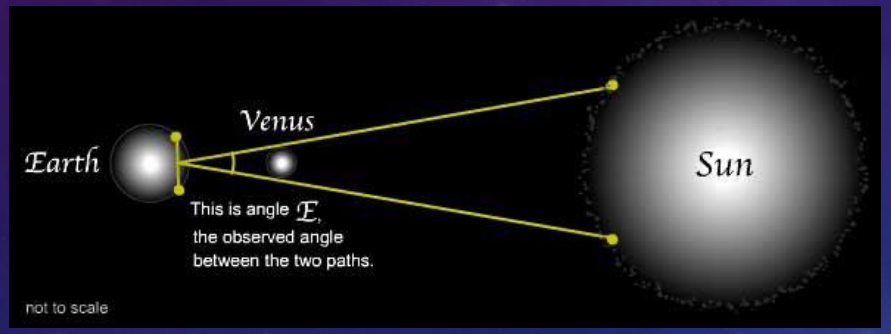


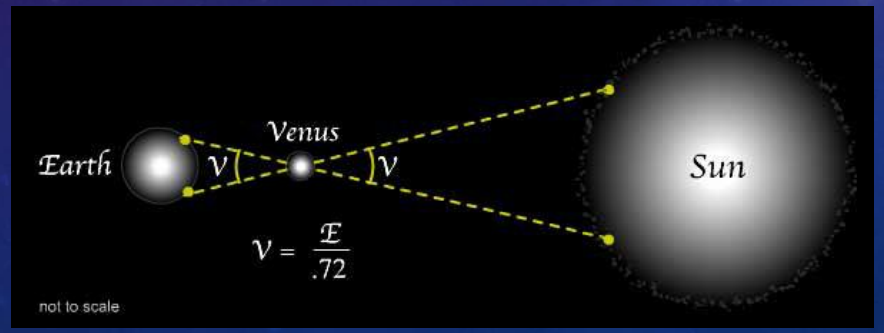


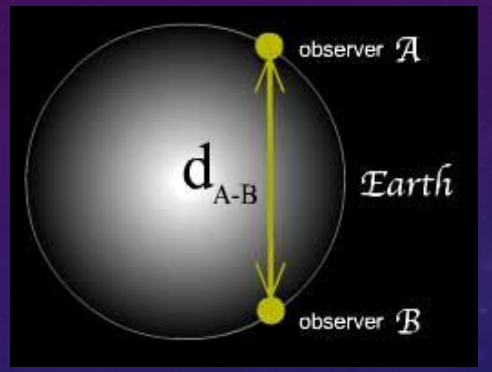
# ASTRONOMSKA JEDINICA

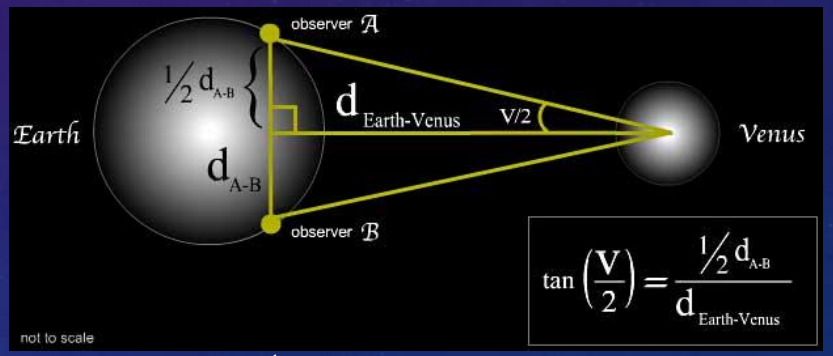



Rastojanje Sunce-Venera (0.72 AU)




$$d_{EV} = \frac{\frac{1}{2} d_{AB}}{\tan(\frac{v}{2})} \Rightarrow d_{EV} = \frac{d_{AB}}{\tan(v)}$$

$$d_{ES} = d_{EV} + d_{VS}$$

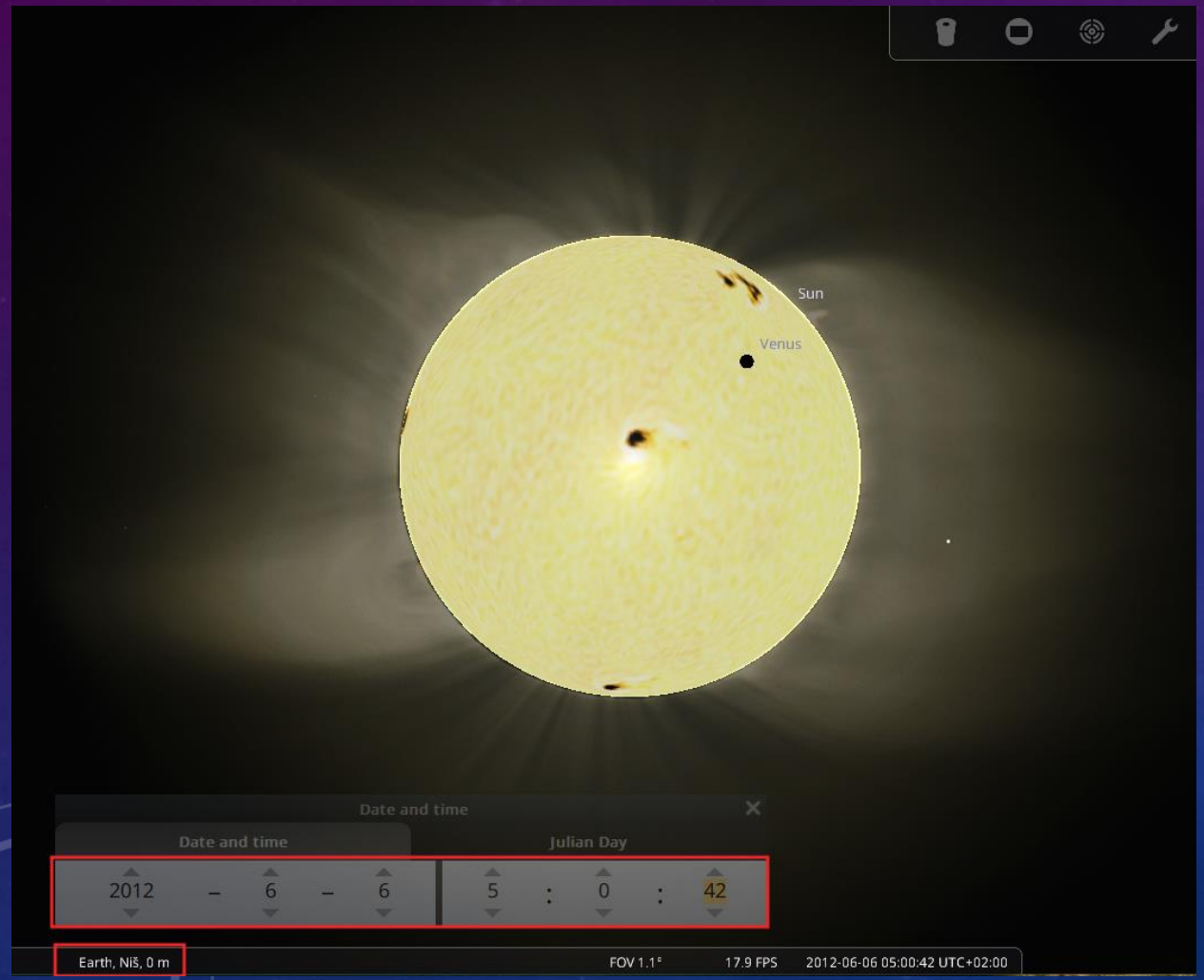
$$d_{EV} = 0.28 d_{ES}$$

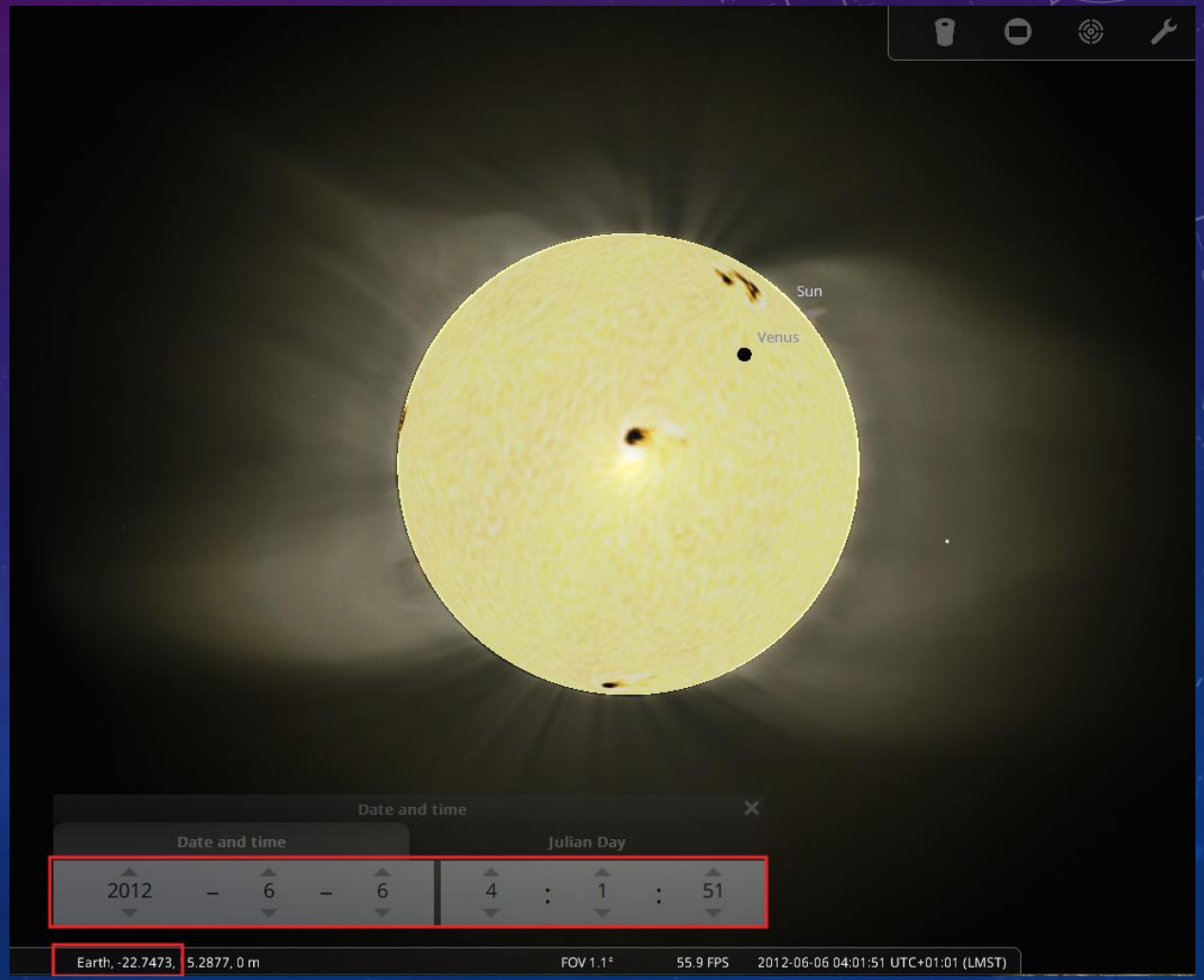
# KAKO?

- Merimo paralaksu $\pi_m$ [lučne sekunde]
- *B* – rastojanje između dve lokacije [km]
  - Meri se u odnosu na pravac normalan na pravu centar Zemlje – Sunca
- $D_{ev}$ - rastojanje Zemlja – Venera [AU]
- $D_{es}$ - rastojanje Zemlja – Sunce [AU]
- Računamo A [km]

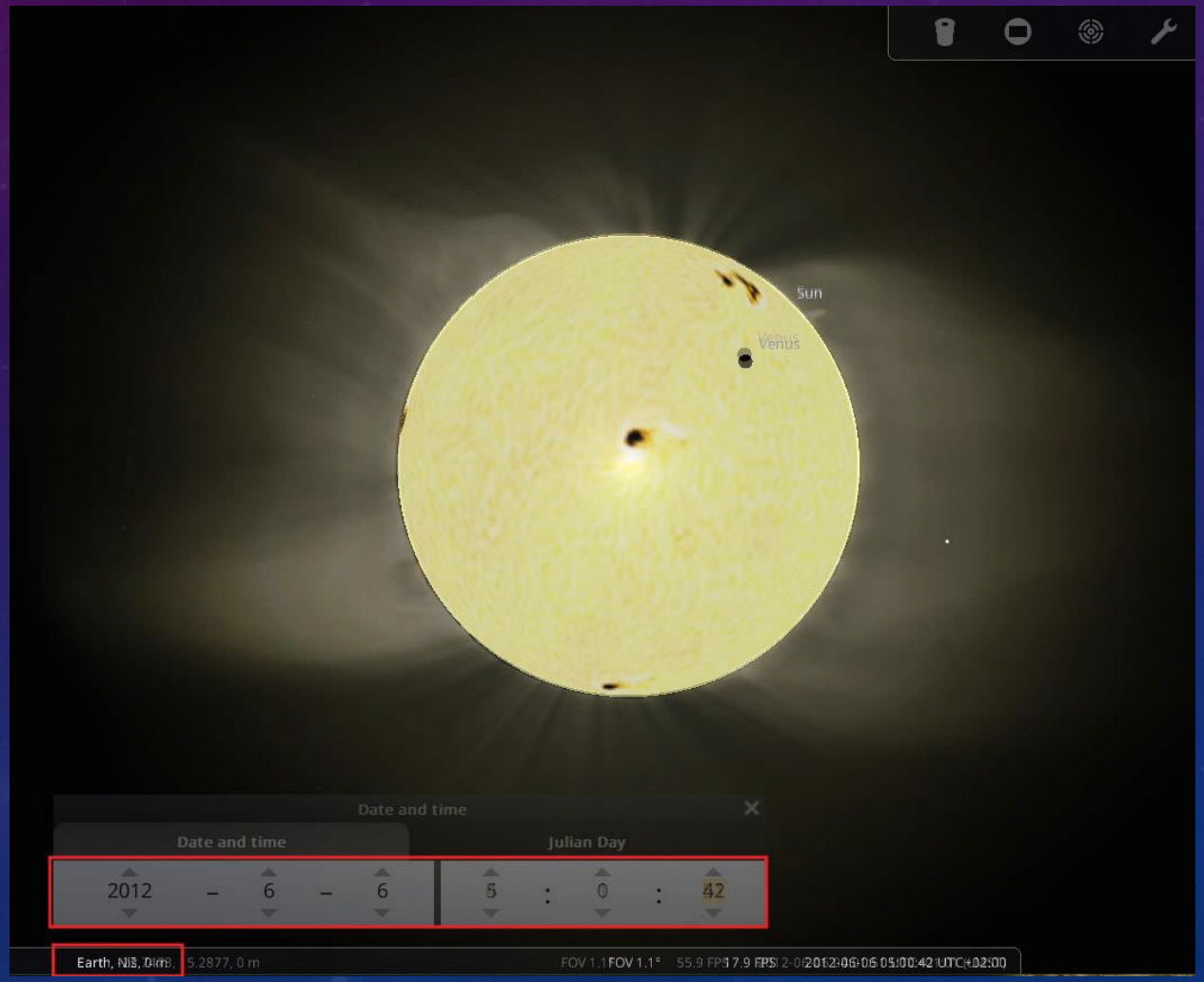
$$A = \frac{B \cdot (D_{es} - D_{ev}) \cdot 206265}{\pi_m \cdot D_{ev} \cdot D_{es}}$$

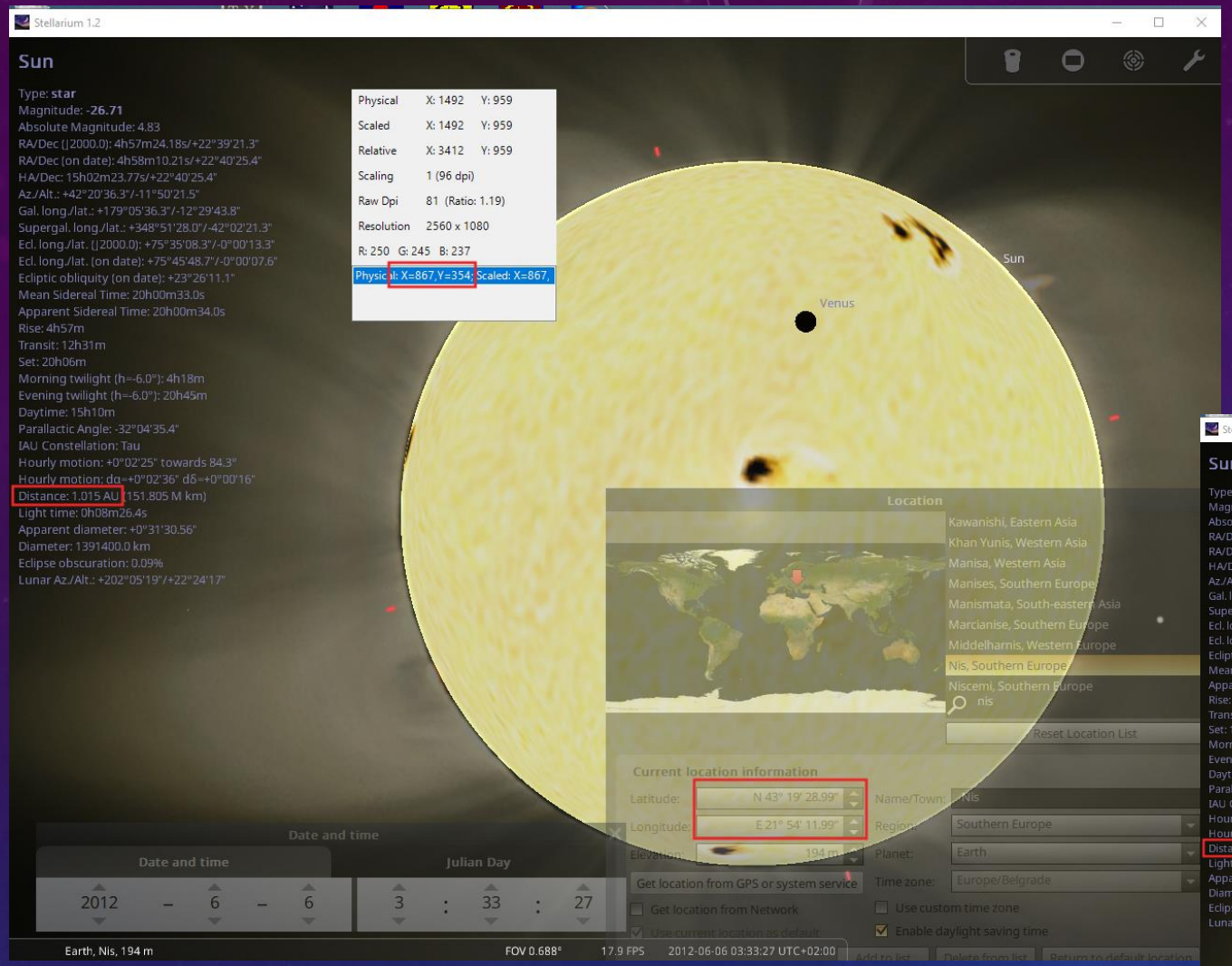
# DA PROBAMO ☺

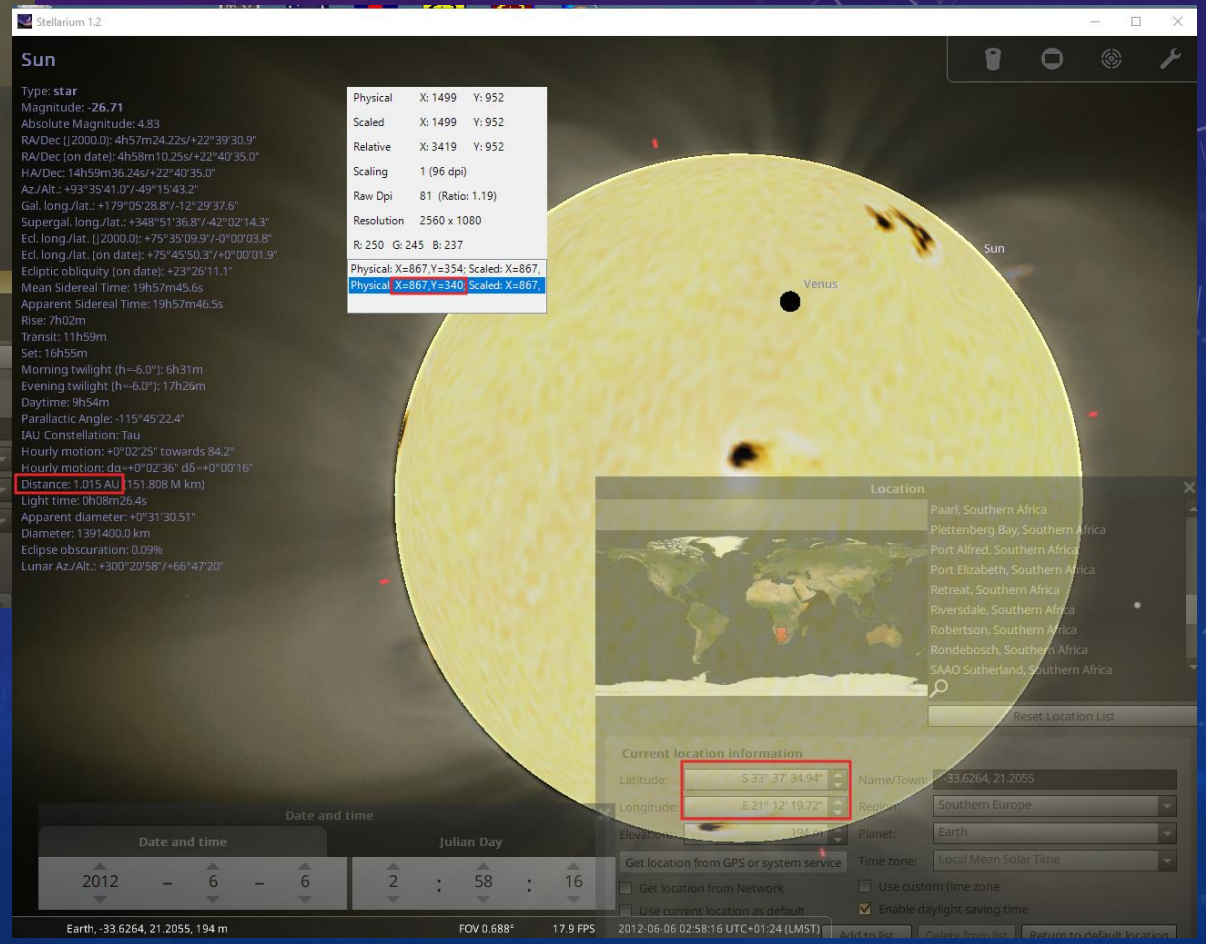

Sun
Venus
Date and time
Date and time
Julian Day
2012 – 6 – 6
5 : 0 : 42
Earth, Niš, 0 m
FOV 1.1°
17.9 FPS
2012-06-06 05:00:42 UTC+02:00

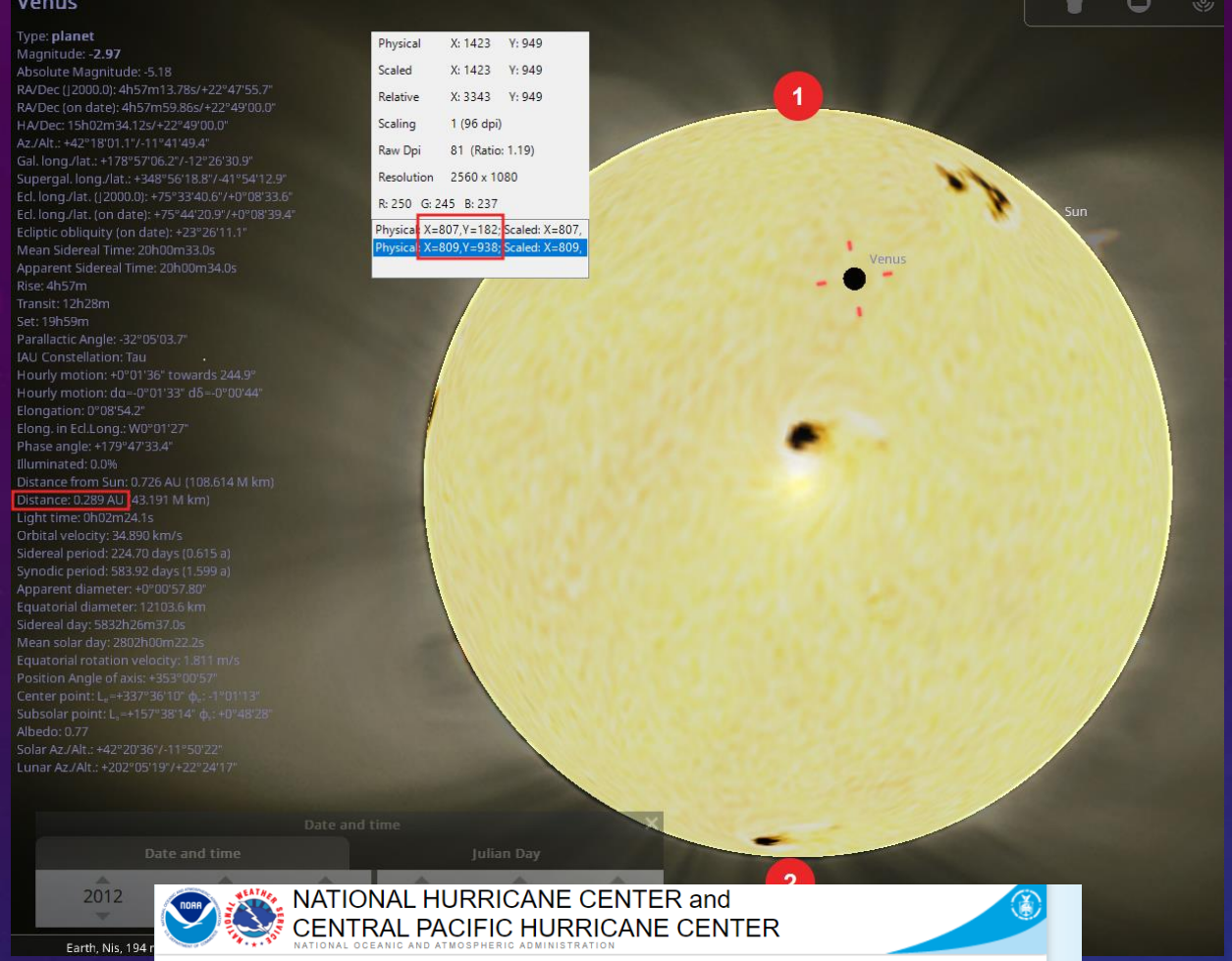

Sun
Venus
Date and time
Date and time
Julian Day
2012 – 6 – 6
4 : 1 : 51
Earth, -22.7473, 5.2877, 0 m
FOV 1.1°
55.9 FPS
2012-06-06 04:01:51 UTC+01:01 (LMST)

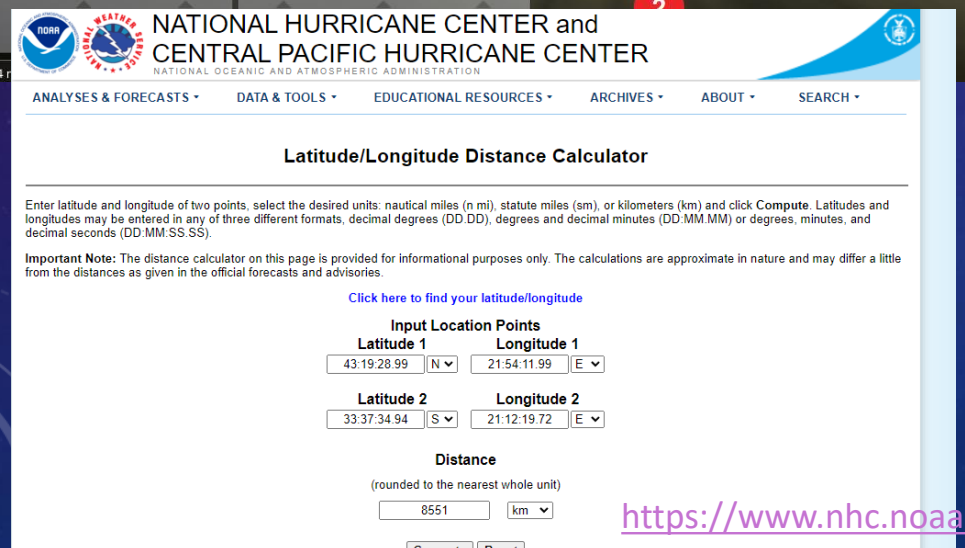
# DA PROBAMO ☺




Stellarium 1.2
Sun
Type: star
Magnitude: -26.71
Absolute Magnitude: 4.83
RA/Dec (J2000.0): 4h57m24.18s/+22°39'21.3"
RA/Dec (on date): 4h58m10.21s/+22°40'25.4"
HA/Dec: 15h02m23.77s/+22°40'25.4"
Az./Alt.: +42°20'36.3"/-11°50'21.5"
Gal. long./lat.: +179°05'36.3"/-12°29'43.8"
Supergal. long./lat.: +348°51'28.0"/-42°02'21.3"
Ecl. long./lat. (J2000.0): +75°35'08.3"/-0°00'13.3"
Ecl. long./lat. (on date): +75°45'48.7"/-0°00'07.6"
Ecliptic obliquity (on date): +23°26'11.1"
Mean Sidereal Time: 20h00m33.0s
Apparent Sidereal Time: 20h00m34.0s
Rise: 4h57m
Transit: 12h31m
Set: 20h06m
Morning twilight (h=-6.0°): 4h18m
Evening twilight (h=-6.0°): 20h45m
Daytime: 15h10m
Parallactic Angle: -32°04'35.4"
IAU Constellation: Tau
Hourly motion: +0°02'25" towards 84.3°
Hourly motion: dα=+0°02'36" dδ=+0°00'16"
Distance: 1.015 AU (151.805 M km)
Light time: 0h08m26.4s
Apparent diameter: +0°31'30.56"
Diameter: 1391400.0 km
Eclipse obscuration: 0.09%
Lunar Az./Alt.: +202°05'19"/+22°24'17"
Physical X: 1492 Y: 959
Scaled X: 1492 Y: 959
Relative X: 3412 Y: 959
Scaling 1 (96 dpi)
Raw Dpi 81 (Ratio: 1.19)
Resolution 2560 x 1080
R: 250 G: 245 B: 237
Physical: X=867,Y=354; Scaled: X=867,
Sun
Venus
Location
Kawanishi, Eastern Asia
Khan Yunis, Western Asia
Manisa, Western Asia
Manises, Southern Europe
Manismata, South-eastern Asia
Marcianise, Southern Europe
Middelharnis, Western Europe
Nis, Southern Europe
Niscemi, Southern Europe
nis
Reset Location List
Current location information
Latitude: N 43° 19' 28.99"
Longitude: E 21° 54' 11.99"
Name/Town: Nis
Region: Southern Europe
Planet: Earth
Time zone: Europe/Belgrade
Get location from GPS or system service
Get location from Network
Use custom time zone
Use current location as default
Enable daylight saving time
Date and time
Date and time
2012 - 6 - 6
Julian Day
3 : 33 : 27
Earth, Nis, 194 m
FOV 0.688°
17.9 FPS
2012-06-06 03:33:27 UTC+02:00


Stellarium 1.2
Sun
Type: star
Magnitude: -26.71
Absolute Magnitude: 4.83
RA/Dec (J2000.0): 4h57m24.22s/+22°39'30.9"
RA/Dec (on date): 4h58m10.25s/+22°40'35.0"
HA/Dec: 14h59m36.24s/+22°40'35.0"
Az./Alt.: +93°35'41.0"/-49°15'43.2"
Gal. long./lat.: +179°05'28.8"/-12°29'37.6"
Supergal. long./lat.: +348°51'36.8"/-42°02'14.3"
Ecl. long./lat. (J2000.0): +75°35'09.9"/-0°00'03.8"
Ecl. long./lat. (on date): +75°45'50.3"/+0°00'01.9"
Ecliptic obliquity (on date): +23°26'11.1"
Mean Sidereal Time: 19h57m45.6s
Apparent Sidereal Time: 19h57m46.5s
Rise: 7h02m
Transit: 11h59m
Set: 16h55m
Morning twilight (h=-6.0°): 6h31m
Evening twilight (h=-6.0°): 17h26m
Daytime: 9h54m
Parallactic Angle: -115°45'22.4"
IAU Constellation: Tau
Hourly motion: +0°02'25" towards 84.2°
Hourly motion: dα=+0°02'36" dδ=+0°00'16"
Distance: 1.015 AU (151.808 M km)
Light time: 0h08m26.4s
Apparent diameter: +0°31'30.51"
Diameter: 1391400.0 km
Eclipse obscuration: 0.09%
Lunar Az./Alt.: +300°20'58"/+66°47'20"
Physical X: 1499 Y: 952
Scaled X: 1499 Y: 952
Relative X: 3419 Y: 952
Scaling 1 (96 dpi)
Raw Dpi 81 (Ratio: 1.19)
Resolution 2560 x 1080
R: 250 G: 245 B: 237
Physical: X=867,Y=354; Scaled: X=867,
Physical: X=867,Y=340; Scaled: X=867,
Sun
Venus
Location
Paarl, Southern Africa
Plettenberg Bay, Southern Africa
Port Alfred, Southern Africa
Port Elizabeth, Southern Africa
Retreat, Southern Africa
Riversdale, Southern Africa
Robertson, Southern Africa
Rondebosch, Southern Africa
SAAO Sutherland, Southern Africa
Reset Location List
Current location information
Latitude: S 33° 37' 34.94"
Longitude: E 21° 12' 19.72"
Name/Town: -33.6264, 21.2055
Region: Southern Europe
Planet: Earth
Time zone: Local Mean Solar Time
Get location from GPS or system service
Get location from Network
Use custom time zone
Use current location as default
Enable daylight saving time
Date and time
Date and time
2012 - 6 - 6
Julian Day
2 : 58 : 16
Earth, -33.6264, 21.2055, 194 m
FOV 0.688°
17.9 FPS
2012-06-06 02:58:16 UTC+01:24 (LMST)




https://www.nhc.noaa.gov/gccalc.shtml

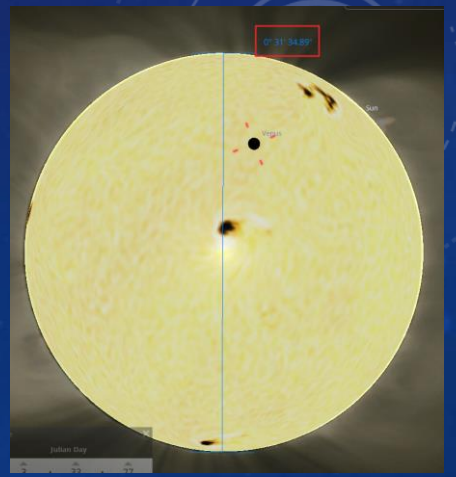
$$A = \frac{B \cdot (D_{es} - D_{ev}) \cdot 206265}{\pi_m \cdot D_{ev} \cdot D_{es}}$$

$$B = 8551 \text{ km}$$
$$D_{es} = 1.015 \text{ AU}$$
$$D_{ev} = 0.289 \text{ AU}$$

$$\Delta Y_v = 354 - 340 = 14$$
$$\Delta Y_s = 938 - 182 = 745$$

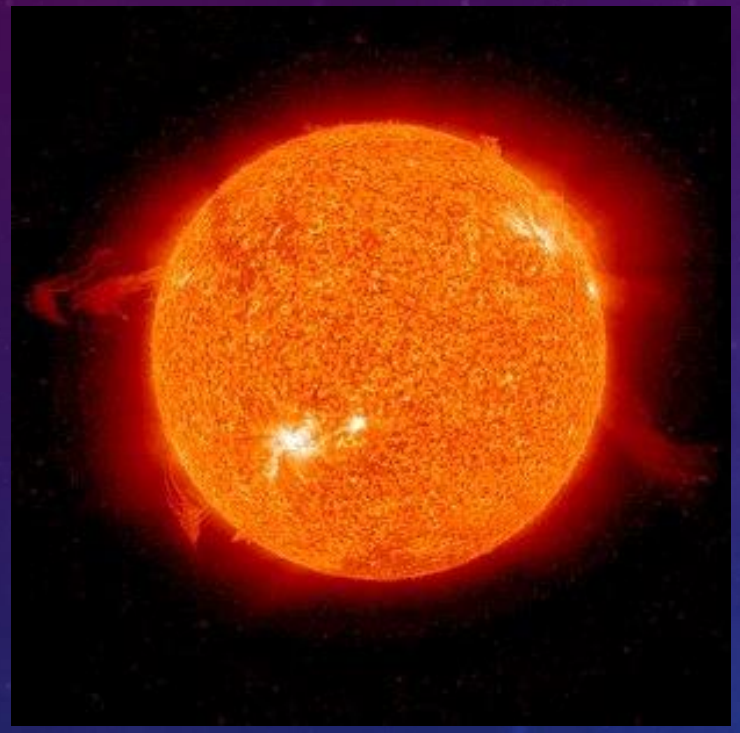
$$\pi_m = 0.0188 \text{ prečnika Sunca}$$
Prečnik Sunca 1894.99 ″
$$\pi_m = 35.626$$

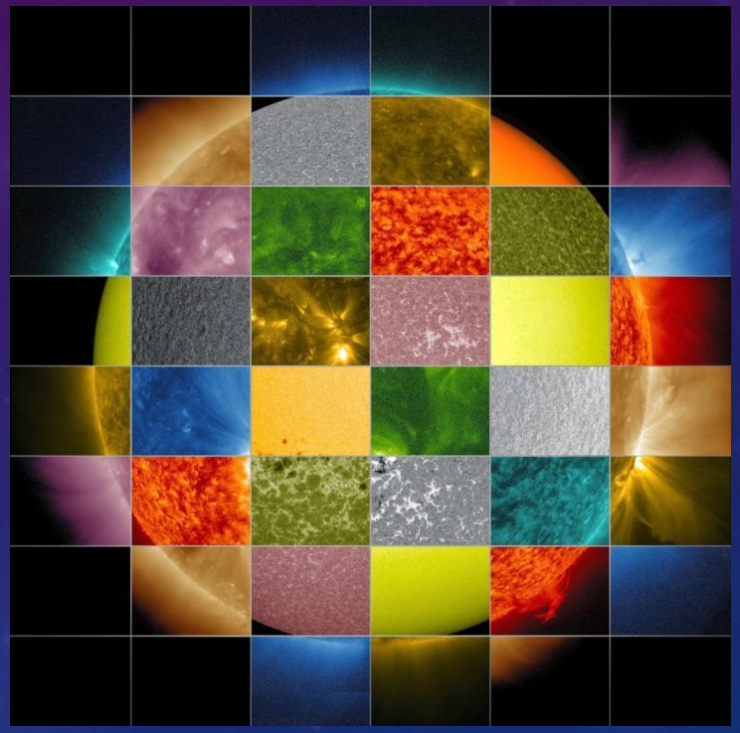
$$A = 122\,532\,250.89 \text{ km}$$



# ROTACIJA SUNCA I DINAMO EFEKAT

# SUNCE U SVIM „BOJAMA“





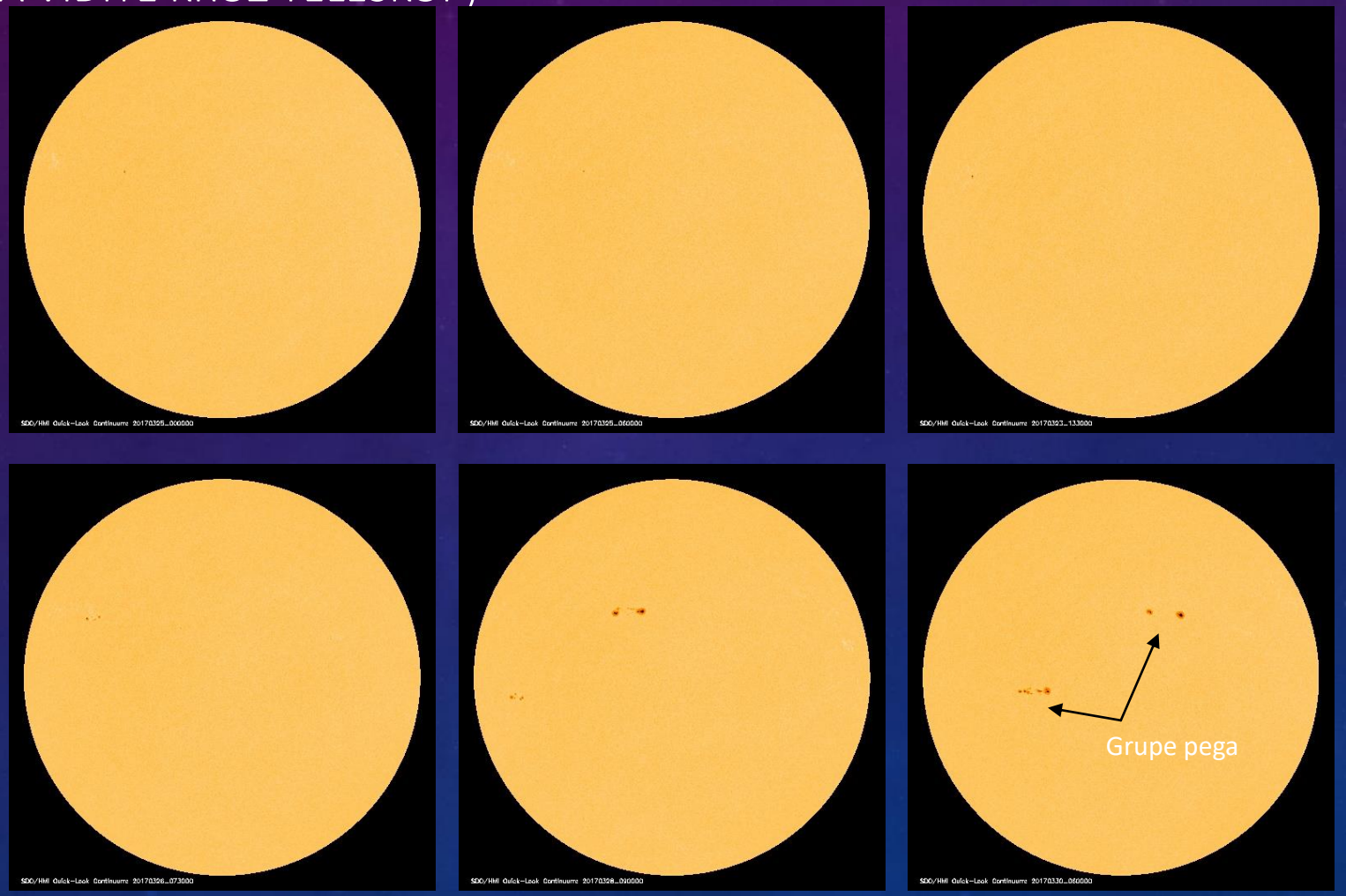
- Golim okom vidimo samo sjajni žuti disk, kroz specijalni teleskop (npr. *Lunt*) vidimo nešto nalik slici levo.
- Astronomi, različitim instrumentima, vide Sunce kao na slici desno (u različitim delovima spektra)

# KAKO BEZBEDNO POSMATRATI SUNCE?

- **Solarni teleskop *Lunt*** sa specijalnim filterima (H-alfa) koji omogućavaju bezbedno posmatranje Sunca i izdvajanje pojedinačnih detalja na površini i u atmosferi naše zvezde (*levo*)
- **Indirektna metoda** – projektovanjem lika Sunca kroz bilo koji teleskopom i posmatranje pega na papiru – zaklonu (*desno*)

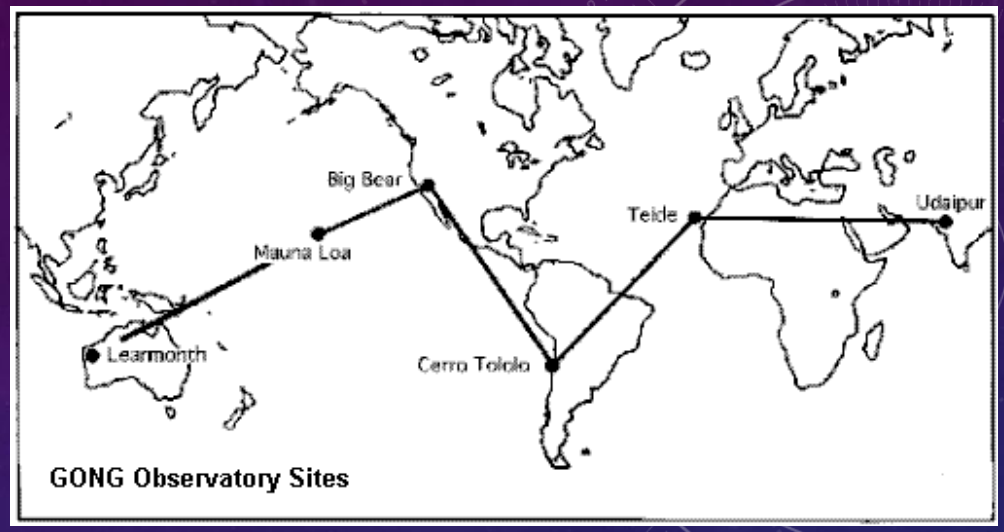
# SUNCE, DANAS, JUČE…

## (ŠTA MOŽETE DA VIDITE KROZ TELESKOP)



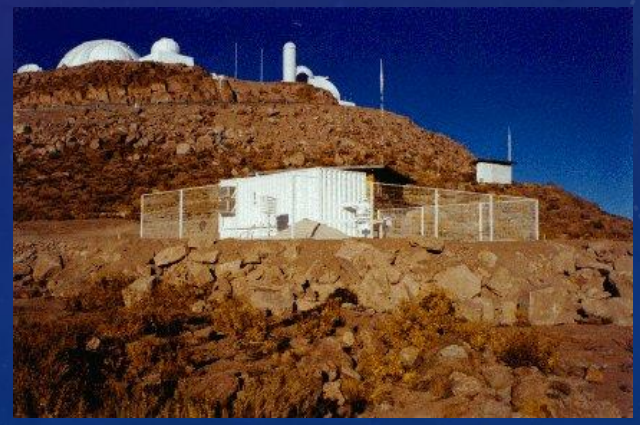

Fotografije: SOHO

# TELESKOPI ŠIROM SVETA

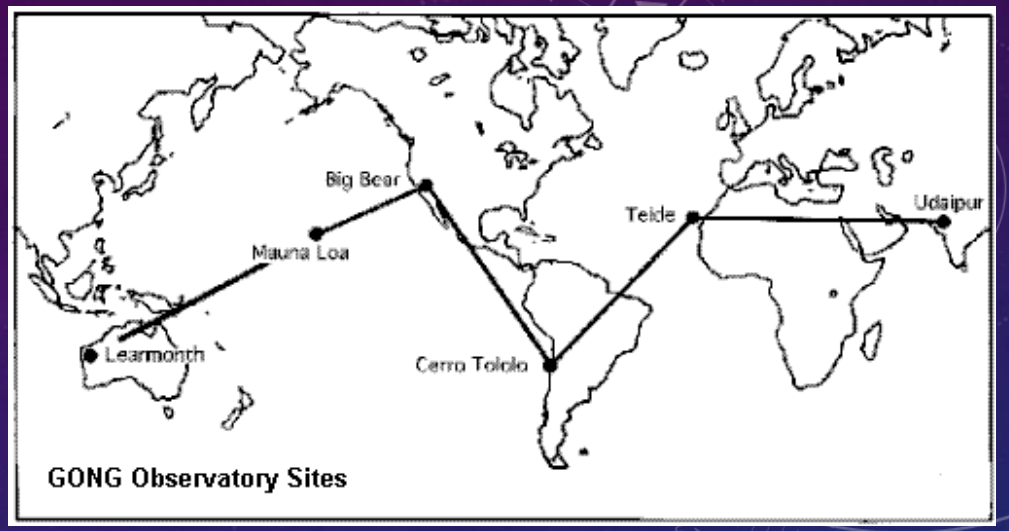



- 1995, National Solar Observatory – mreža teleskopa
  - Upravlja Global Oscillation Network Group (GONG), Tucson, **Arizona**
  - Kad Sunce zađe na jednom teleskopu, na drugom je visoko iznad horizonta
  - Big Bear, **California**; MauLoa, **Hawaii**; Learmonth, **Australia**; Udiapur, **India**; El Teide, **Tenerife** (The Canary Islands, Atlantic Ocean) i Cerro Tololo, **Chile**.

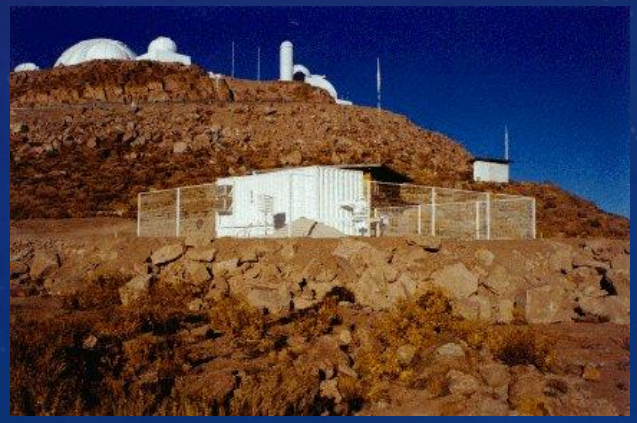
https://gong.nso.edu/



# GONG



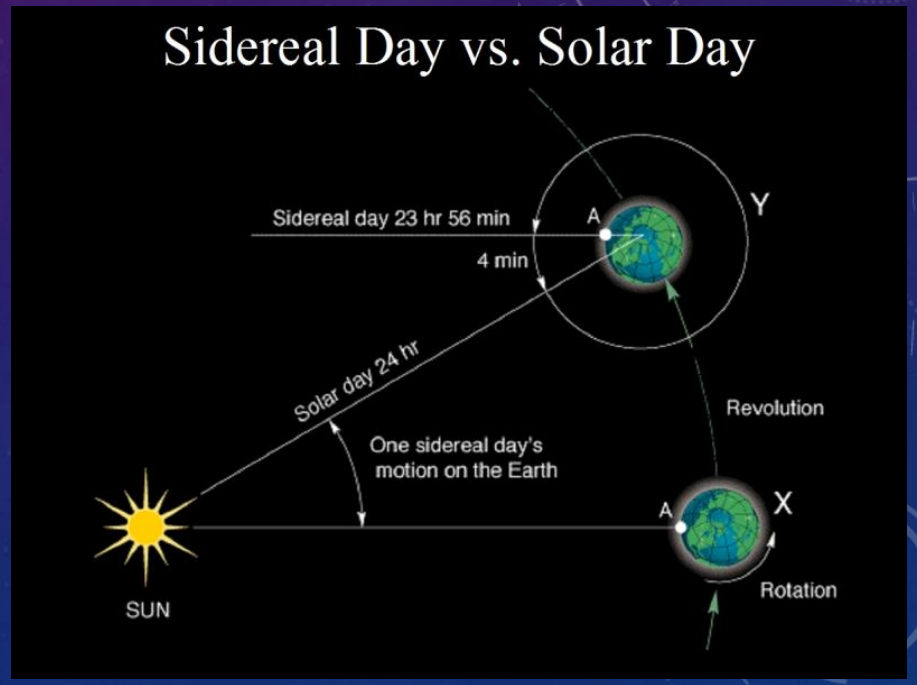

- Robotizovani teleskopi, snimaju Sunce svakog minuta
- CLEA – 368 fotografija, od 1. januara 2002 do 30. aprila 2002
- Ukupno 3600 fotografija dnevno, vežba 3 dnevno
- Digitalno obrađene fotografije; 860x860px, sever – gore
- Disk Sunca – 720px, 1px - 2,5 lučnih sekundi, 1800 km (centar)
  - Na 95% rastojanja do ivice 1px – 6000 km



https://gong.nso.edu/

# MERENJE PERIODA ROTACIJE

- Najlakši način – čekamo da se pega ponovo pojavi, ali…
  - Da li žive dovoljno dugo? Kako da je prepoznamo?
  - Možda ne „pogodimo“ da je slikamo? Itd …
- Siderički i sinodički period
  - **Sinodički period** – vidimo sa Zemlje
  - Nije pravi period, rotira i Zemlja
  - Pravi period – **siderički** (period u odnosu na odeljene zvezde)
  - **Sinodički > sideričkog** (Sunce mora da rotira duže da „stigne“ Zemlju)



# SINODIČKI I SIDERIČKI PERIOD

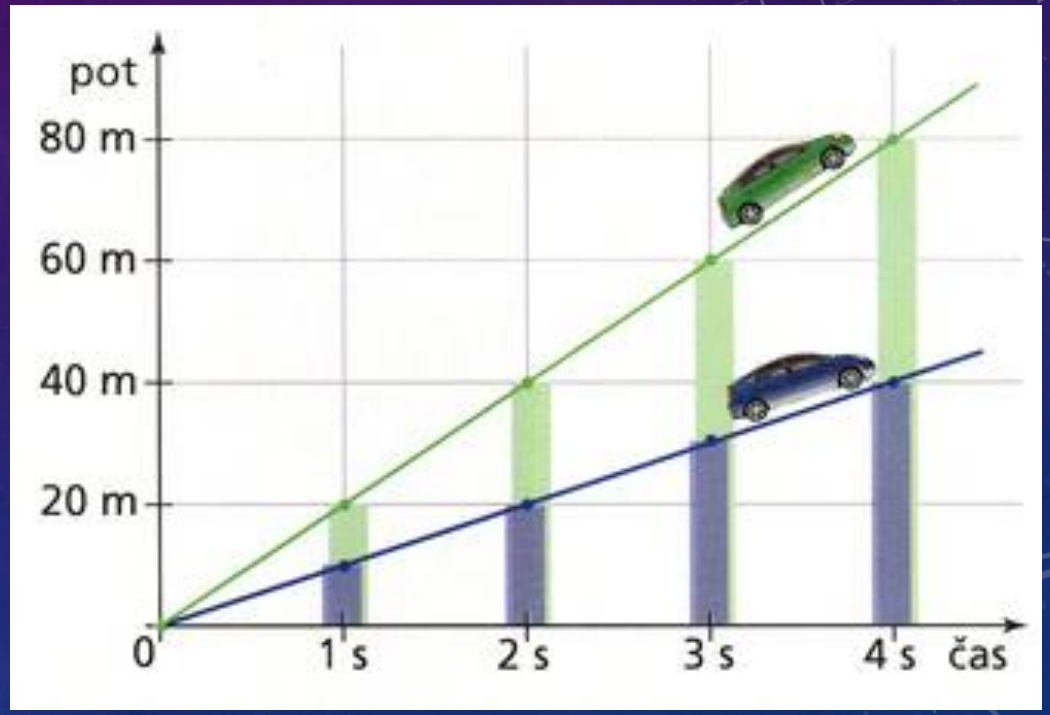
$$v = \frac{s}{t} \Rightarrow t = \frac{s}{v}$$

$$\frac{1}{S} = \frac{1}{P} - \frac{1}{E}$$

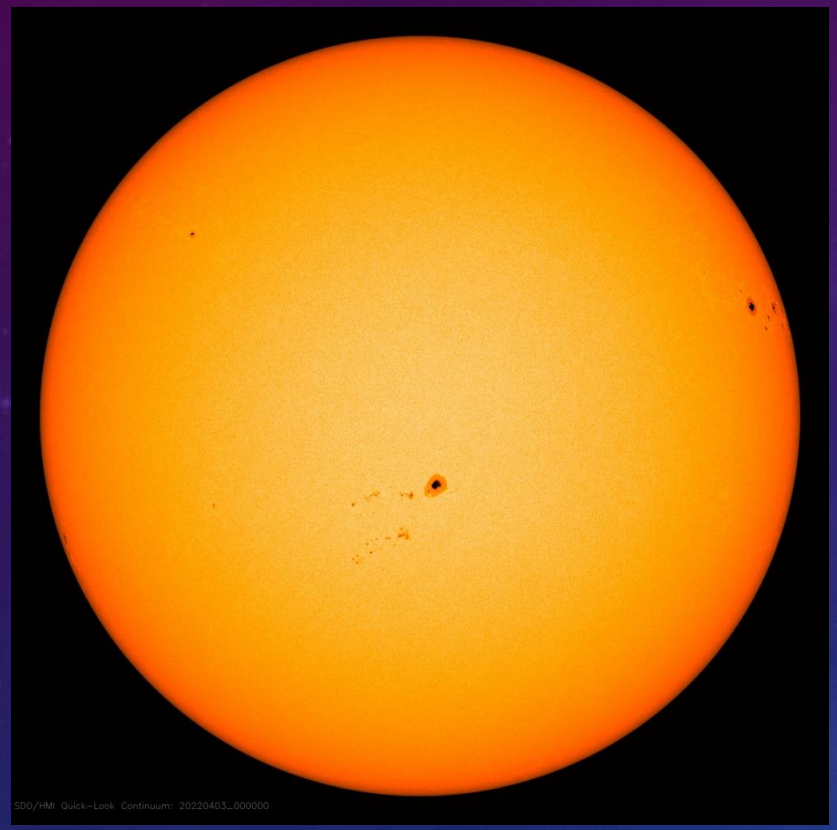
$$\frac{1}{S} = \frac{1}{P} - \frac{1}{365.25} \Rightarrow P = \frac{365.25 * S}{S + 365.25}$$

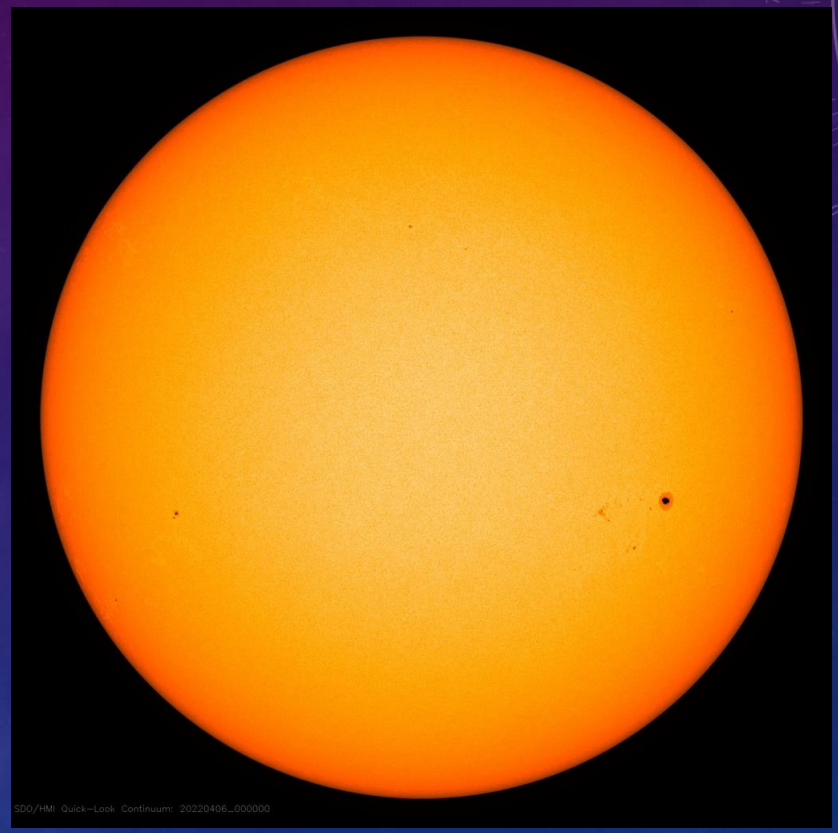
*P* – siderički period
*S* – sinodički period

$$S_{dani} = \frac{360^o}{nagib_{(stepen/danu)}}$$



# ROTACIJA SUNCA

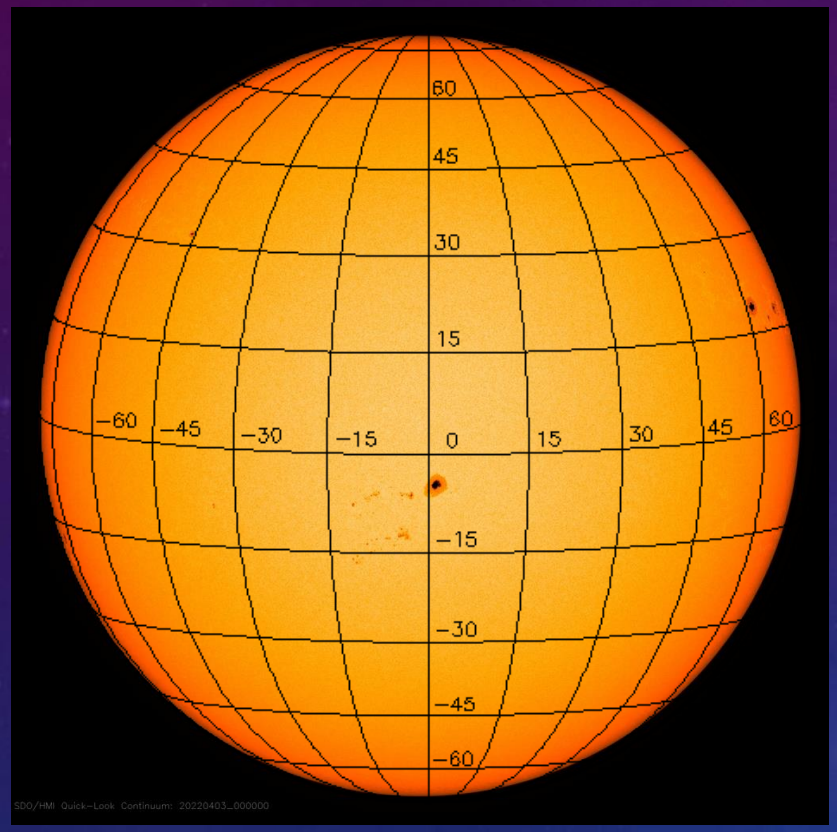


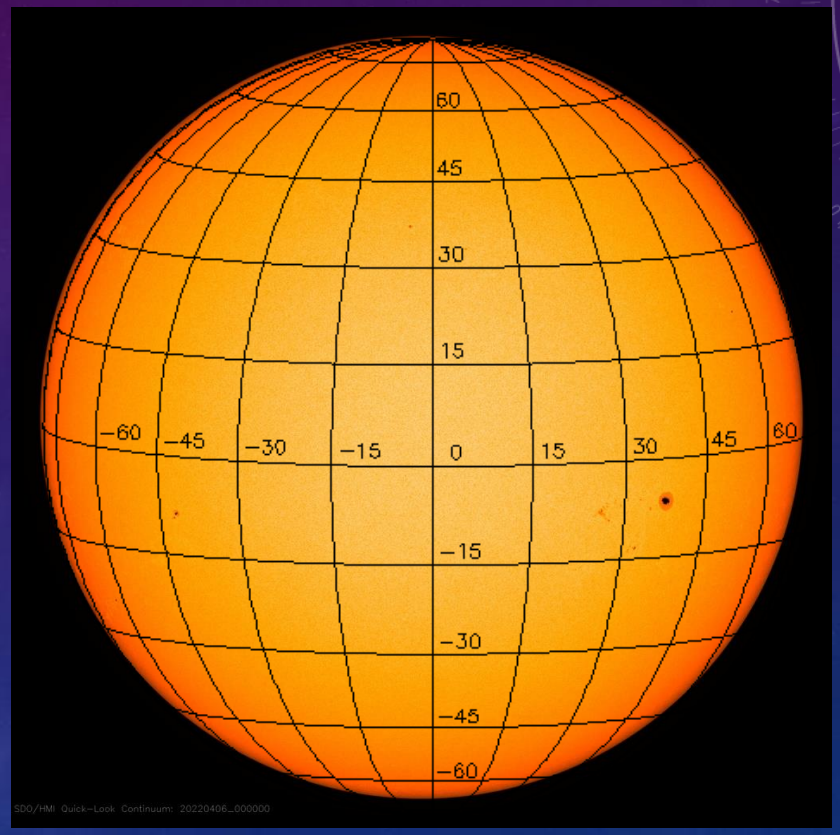




- Praćenjem pega i određivanjem njihove brzine, na osnovu upoređivanja njihovog položaja na fotografijama izračunava se period rotacije Sunca oko ose.
- Oblasti oko ekvatora rotiraju brže od oblasti blizu polova:
  - Diferencijalna rotacija => Sunce nije čvrsto telo!!!

Fotografije: SOHO

# ROTACIJA SUNCA

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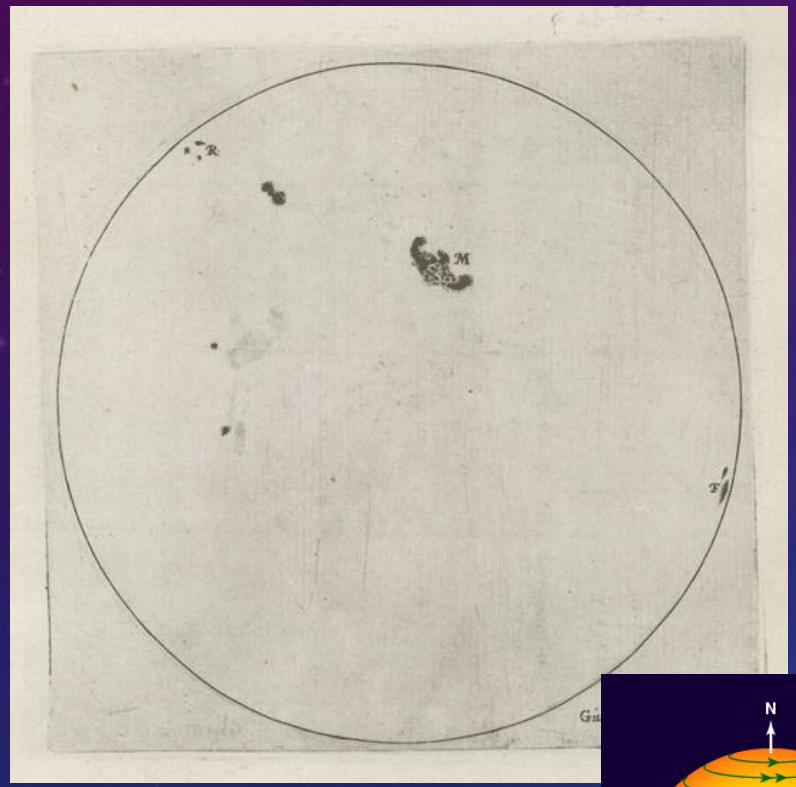
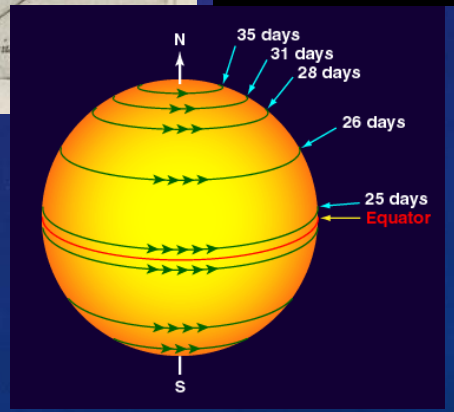
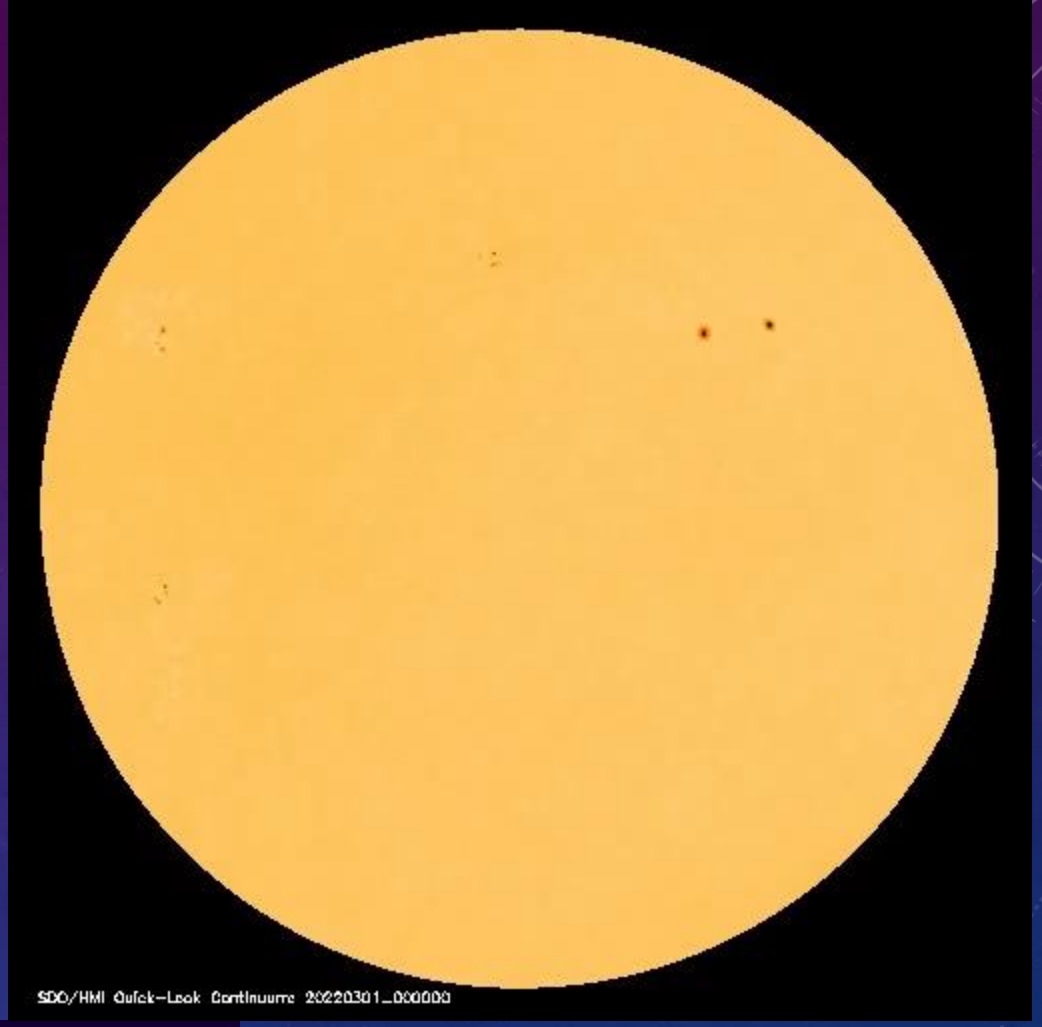
- Oblasti oko ekvatora rotiraju brže od oblasti blizu polova:
  - Diferencijalna rotacija => Sunce nije čvrsto telo!!!

# ROTACIJA SUNCA



Pege na Suncu koje je posmatrao i nacrtao Galileo Galilej




Pege tokom prethodnih par dana, kako ih je snimio satelit SOHO
https://soho.nascom.nasa.gov/

# AD „ALFA“ APLIKACIJA

U procesu kalibracije neophodno je klikom označiti maksimalnu severnu, južnu, isočnu i zapadnu tački Sunca (tj. minimum i maksimum x i y ose). Kalibracija nije neophodna za otografije novijeg datuma ali je potrebna za fotografije gde je crrni okvir značajno veći nego na npr. današnjoj fotografiji.

Kalibracija

Prikažite fotografiju za željeni datum.

Kliknite ovde za izbor datum: Prikaži

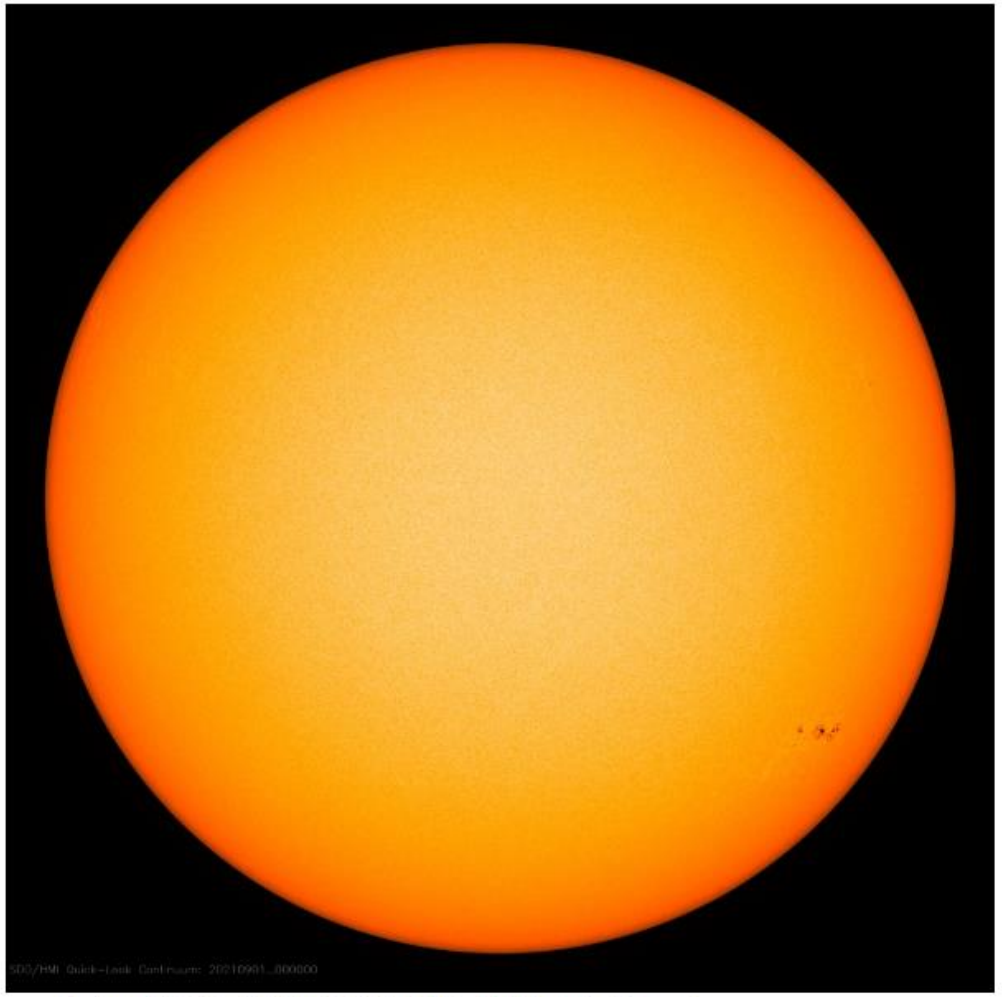
Za izabrani datum prikazaće se fotografija Sunca. Kliknite dva puta na željenu pegu da bi sačuvali njene koordinate. Nakon toga pređite na sledeći dan, ponovo označite istu pegu itd. Postupak nastavite dok je pega vidljiva.

Sačuvaj Dalje Grafik

Snimi i pređi na sledeću

Obriši sačuvano

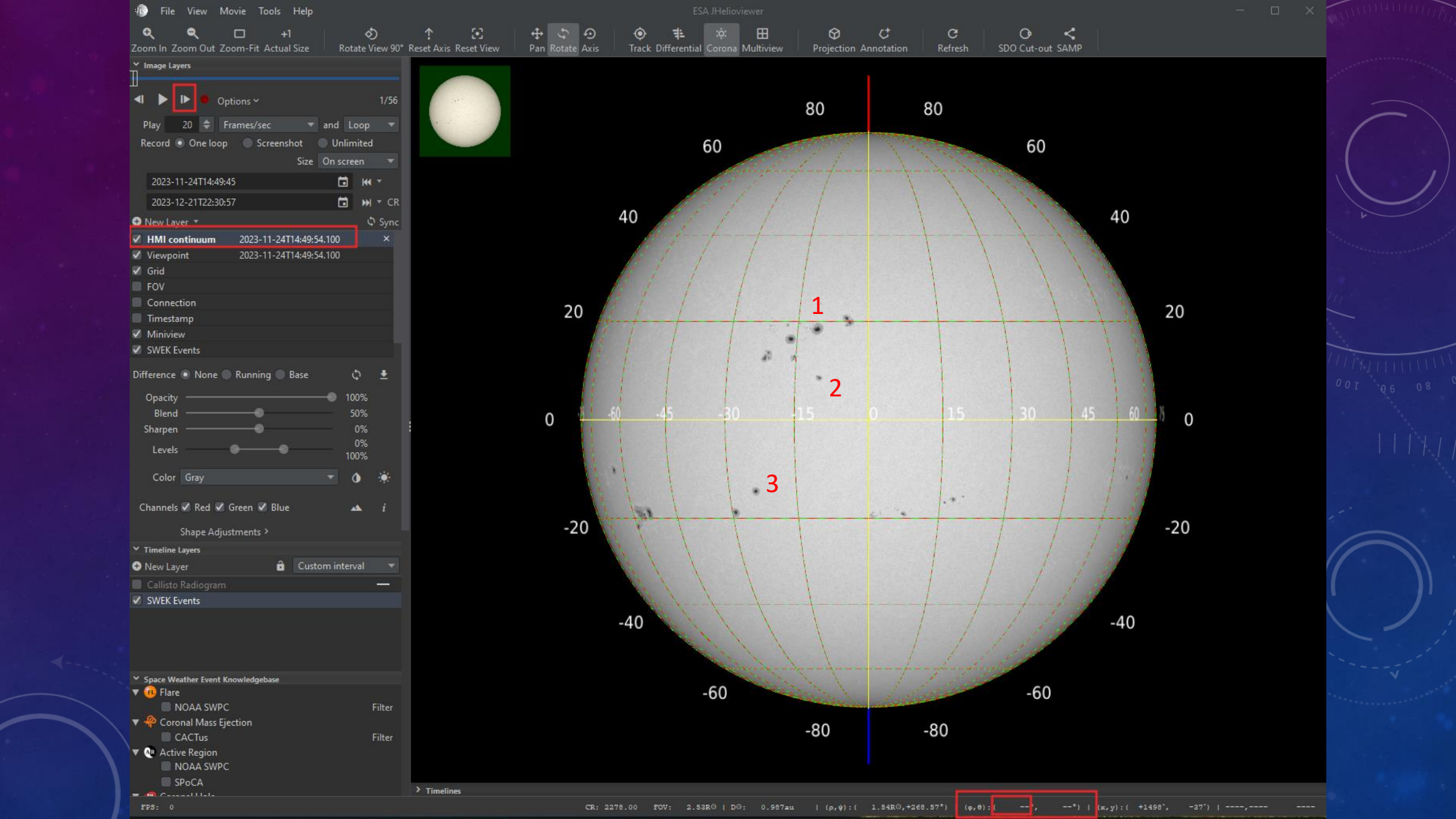
**Pripremili:** Milan Milošević, Irina Cvetković, Petra Nešić i Jovana Stanimirović u okviru projekta *Malim koracima ka astronomiji* koji realizuje AD "Alfa" uz podršku Centra za promociju nauke



Sačuvane koordinate

**Fotografije:** NASA/SDO i AIA, EVE, i HMI naučni timovi

https://alfa.org.rs/sunspots/


File View Movie Tools Help
ESA JHelioviewer
Zoom In Zoom Out Zoom-Fit Actual Size
Rotate View 90° Reset Axis Reset View
Pan Rotate Axis
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Projection Annotation
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Image Layers
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1/56
Play 20 Frames/sec and Loop
Record One loop Screenshot Unlimited
Size On screen
2023-11-24T14:49:45
2023-12-21T22:30:57
CR
New Layer
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Viewpoint 2023-11-24T14:49:54.100
Grid
FOV
Connection
Timestamp
Miniview
SWEK Events
Difference None Running Base
Opacity 100%
Blend 50%
Sharpen 0%
Levels 0%
100%
Color Gray
Channels Red Green Blue
Shape Adjustments
Timeline Layers
New Layer
Custom interval
Callisto Radiogram
SWEK Events
Space Weather Event Knowledgebase
Flare
NOAA SWPC
Filter
Coronal Mass Ejection
CACTus
Filter
Active Region
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3
Timelines
FPS: 0
CR: 2278.00
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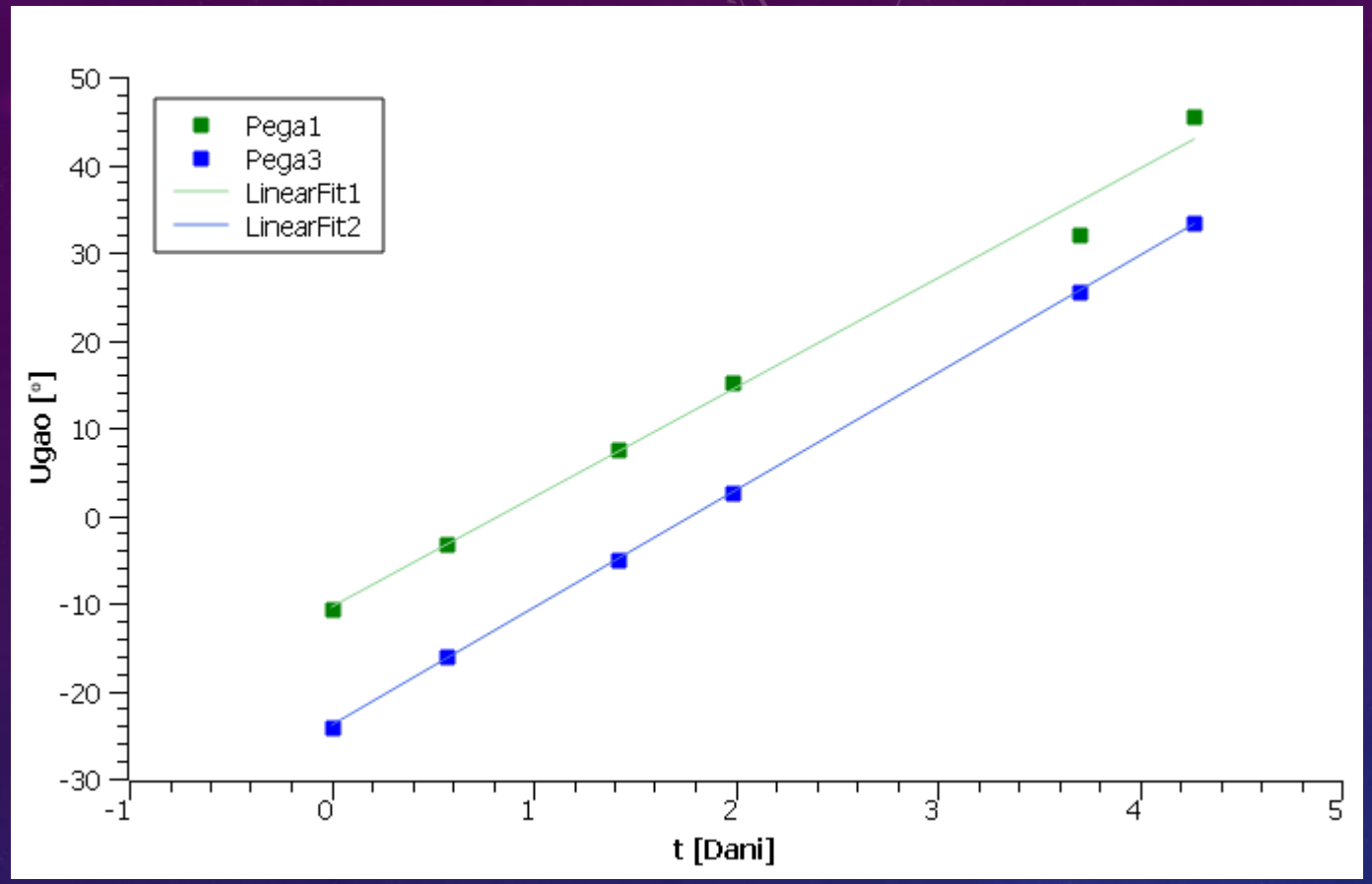
(ρ,ψ):( 1.54R⊙,+268.57°)
(φ,θ):( --°, --°)
(x,y):( +1498", -37")




$$y_1 = k \cdot x_1 + n$$
$$y_2 = k \cdot x_2 + n$$

$$k_1 = 12.48$$
$$k_2 = 13.38$$

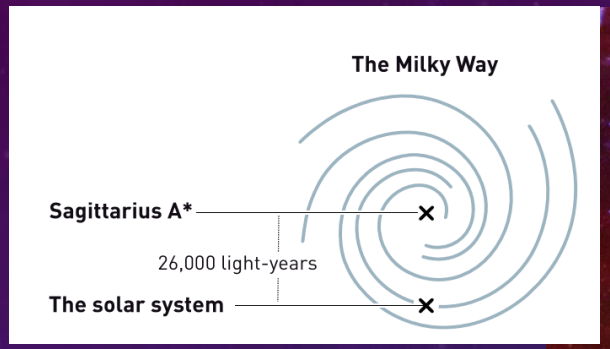
| k | S | P |
|---|---|---|
| 12.48 | 28.84615 | 26.73474 |
| 13.38 | 26.90583 | 25.05982 |

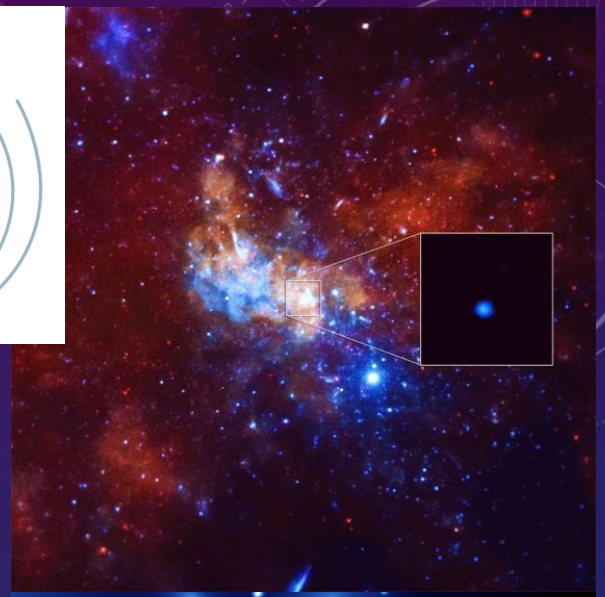
$$S_{dani} = \frac{360^o}{nagib_{(stepen/danu)}}$$

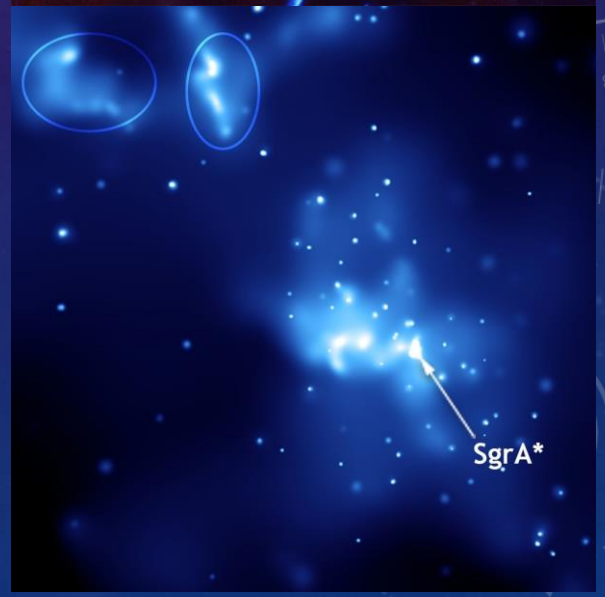
$$P = \frac{365.25 * S}{S + 365.25}$$

# SAVREMENA ASTRONOMIJA

# SMBH U MLEČNOM PUTU
## *Sagitarius A**



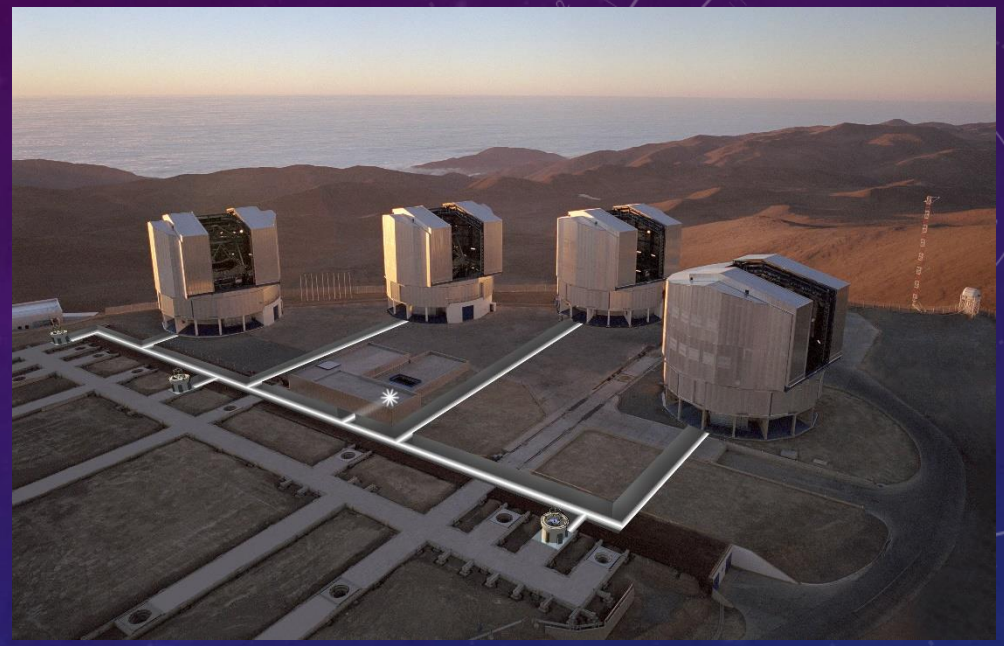



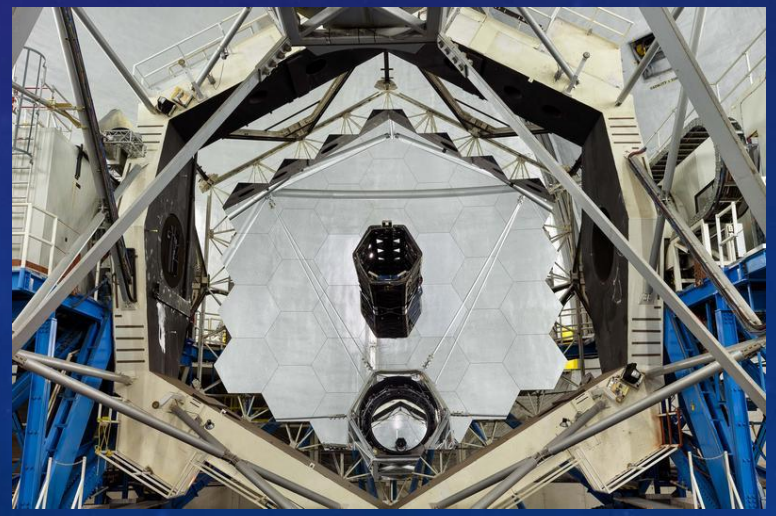



- Jak i kompaktan radio izvor u centru Galaksije
  - Blizu granice sazvežđa Strelac i Škorpija
- Još od otkrića kvazara – pretpostavka SMBH u centru velikih galaksija
  - masa par miliona do nekoliko milijardu masa Sunca
- Centar galaksije Harlow Shapley (pre 100 god)
  - Kasnije pokazano da je to Sag A*
- 1990+ god
  - Projekti R. Gencel i A. Gez – posmatranje orbita zvezda u centru Mlečnog puta

# TELESKOPI

- R. Gencel i grupa
  - New Technology Telescope (La Silla mountain, Čile)
  - Very Large Telescope facility, VLT (Čile)
    - 4 teleskopa, najveći 8 metara (2 puta veći nego NTT)
- A. Genz i grupa
  - Keck opservatorija (Havaji)
    - Oko 10 metara (36 šestougaona segmenta)

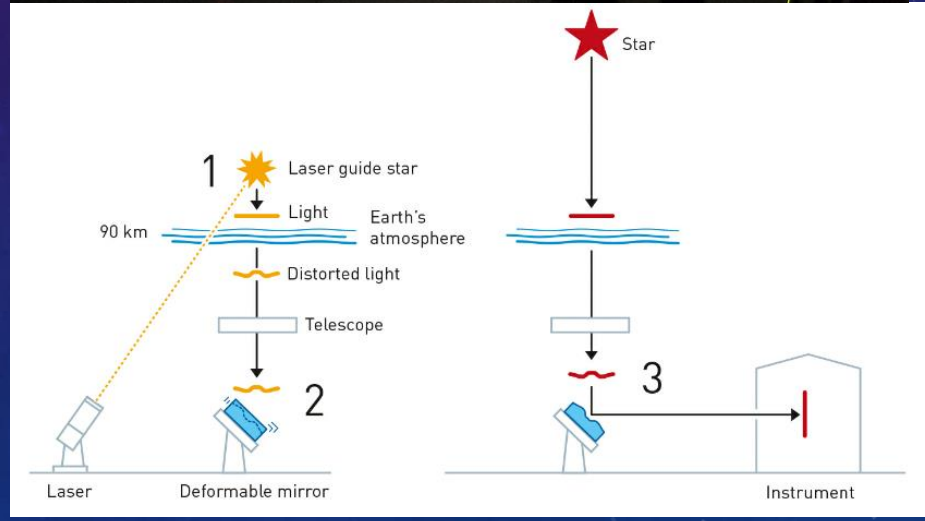




# ZVEZDE PRIČAJU PRIČU

- Ogromni teleskopi ali problem – atmosfera
  - Adaptivna optika
- Istraživači pratili oko 30 sjajnih zvezda
  - 1 svetlosni mesec oko centra
    - velike brzine zvezda
  - Veća rastojanja – stabilnije i „standrdnije“ orbite

- Zvezda S2
  - Period 16 godina – mapirana cela orbita!
  - (Sunce 200 miliona godina)

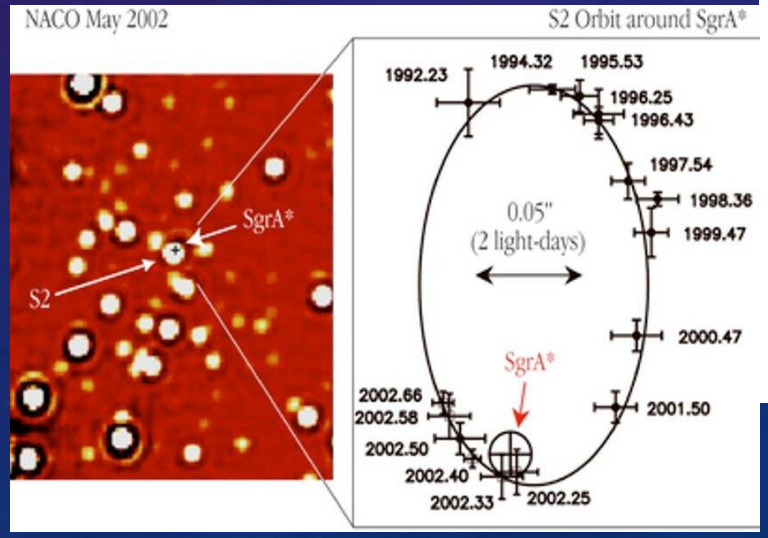


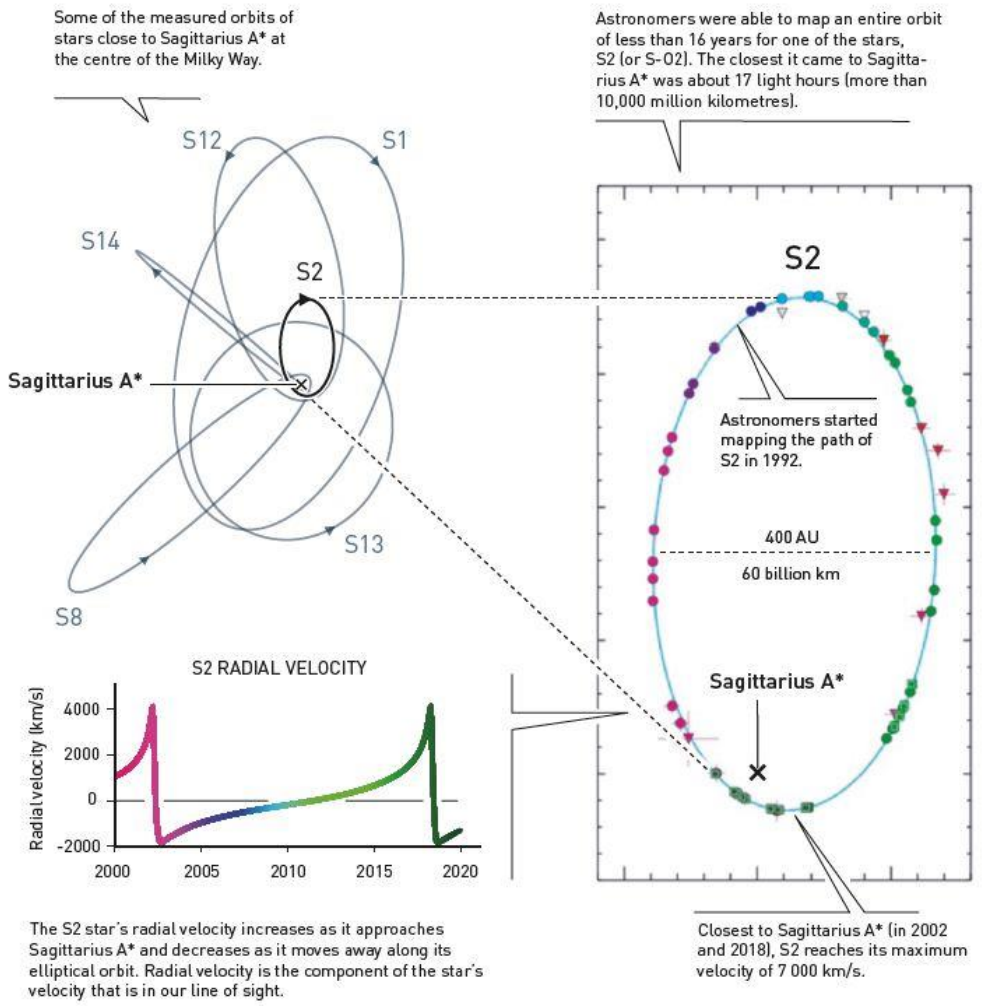

Scientific Background on the Nobel Prize in Physics 2020

# ZVEZDE PRIČAJU PRIČU

- Odlično poklapanje rezultata oba tima
  - SMBH 4 miliona masa Sunca
  - Oblast veličine Sunčevog sistema

- ~~Možda je~~ uskoro stvarno „vidimo“…
  - videli smo je ☺

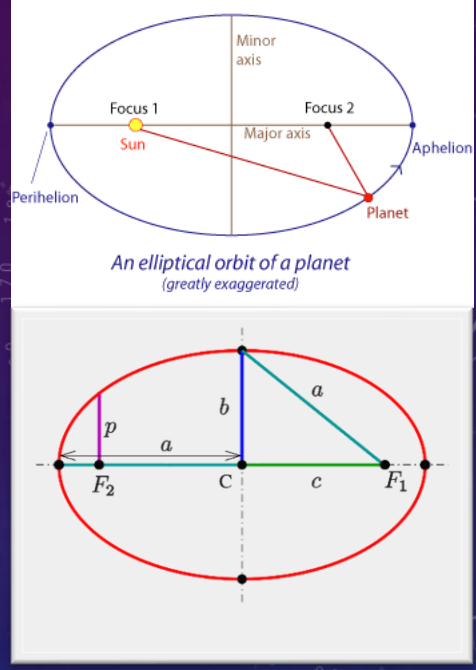


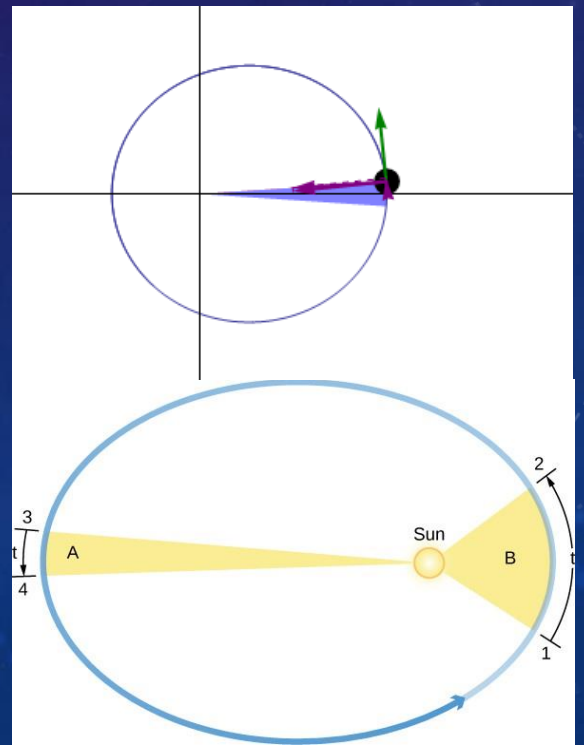



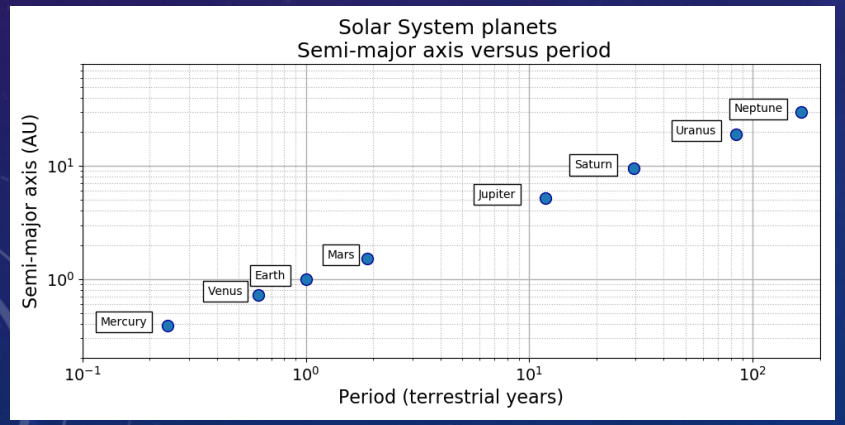

https://www.eso.org/public/images/eso0226c/

# IZRAČUNAJTE MASU SMBH

- Keplerovi zakoni
  1. Planete se oko Sunca kreću po eliptičkim putanjama, u čijoj se zajedničkoj žiži nalazi Sunce
  2. Radijus vektor planete u jednakim vremenskim intervalima opisuje jednake površine
  3. Kvadrati perioda ($P$) obilaska planete oko Sunca srazmerni su kubovima velikih poluosa (a) njihovih putanja



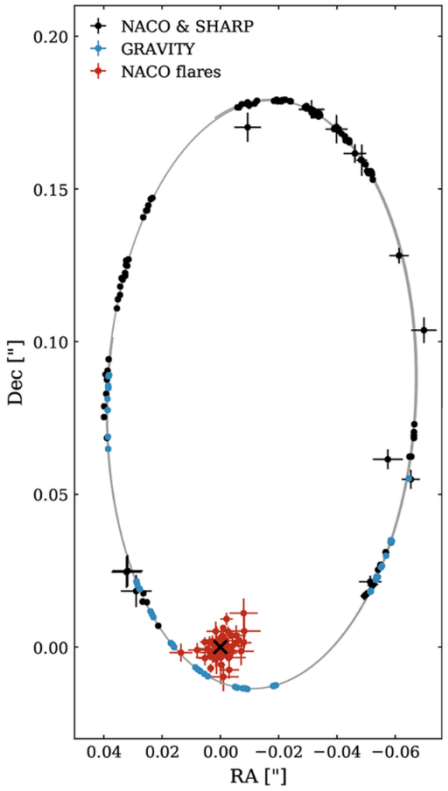




**Modern data (Wolfram Alpha Knowledgebase 2018)**

| Planet | Semi-major axis (AU) | Period (days) | $\frac{R^3}{T^2}$ ($10^{-6}$ AU$^3$/day$^2$) |
|---|---|---|---|
| Mercury | 0.38710 | 87.9693 | 7.496 |
| Venus | 0.72333 | 224.7008 | 7.496 |
| Earth | 1 | 365.2564 | 7.496 |
| Mars | 1.52366 | 686.9796 | 7.495 |
| Jupiter | 5.20336 | 4332.8201 | 7.504 |
| Saturn | 9.53707 | 10775.599 | 7.498 |
| Uranus | 19.1913 | 30687.153 | 7.506 |
| Neptune | 30.0690 | 60190.03 | 7.504 |

# IZRAČUNAJTE MASU SMBH

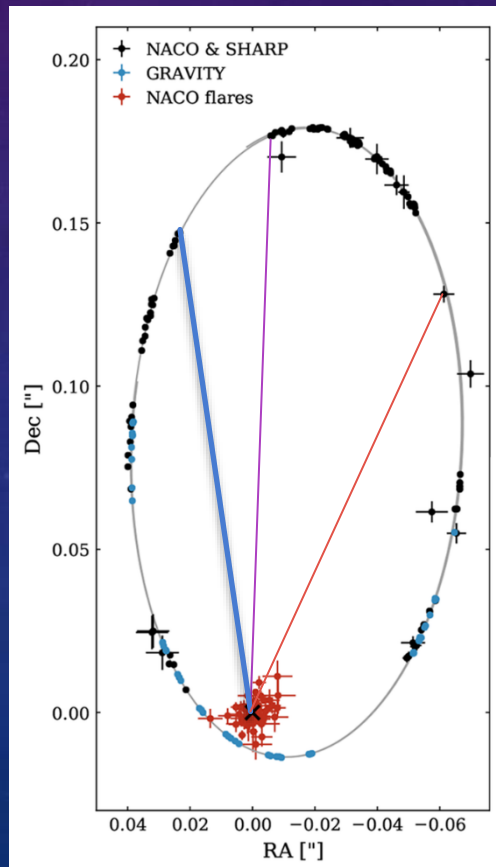
- Levo – koordinate položaja zvezde S2
- Koordinatni početak – centar SMBH
- Kako?
  - Nacrtati tačke (i greške)
  - Nacrtati elipsu najpribližniju merenjima
  - Izmeriti veliku poluosu elipse
    - Arcsec prevesti u svetlosne dane (*ld*), 2 arcsec = 28 ld
    - Izračunati srednju vrednost ☺




| Date (year) | x (arcsec) | dx (arcsec) | y (arcsec) | dy (arcsec) |
|---|---|---|---|---|
| 1992.226 | 0.104 | 0.003 | -0.166 | 0.004 |
| 1994.321 | 0.097 | 0.003 | -0.189 | 0.004 |
| 1995.531 | 0.087 | 0.002 | -0.192 | 0.003 |
| 1996.256 | 0.075 | 0.007 | -0.197 | 0.010 |
| 1996.428 | 0.077 | 0.002 | -0.193 | 0.003 |
| 1997.543 | 0.052 | 0.004 | -0.183 | 0.006 |
| 1998.365 | 0.036 | 0.001 | -0.167 | 0.002 |
| 1999.465 | 0.022 | 0.004 | -0.156 | 0.006 |
| 2000.474 | -0.000 | 0.002 | -0.103 | 0.003 |
| 2000.523 | -0.013 | 0.003 | -0.113 | 0.004 |
| 2001.502 | -0.026 | 0.002 | -0.068 | 0.003 |
| 2002.252 | -0.013 | 0.005 | 0.003 | 0.007 |
| 2002.334 | -0.007 | 0.003 | 0.016 | 0.004 |
| 2002.408 | 0.009 | 0.003 | 0.023 | 0.005 |
| 2002.575 | 0.032 | 0.002 | 0.016 | 0.003 |
| 2002.650 | 0.037 | 0.002 | 0.009 | 0.003 |
| 2003.214 | 0.072 | 0.001 | -0.024 | 0.002 |
| 2003.353 | 0.077 | 0.002 | -0.030 | 0.002 |
| 2003.454 | 0.081 | 0.002 | -0.036 | 0.002 |

# IZRAČUNAJTE MASU SMBH

- Kako?
  - …
  - Odrediti period ($P$)
    - $A_{ell} = ab \cdot \pi$ - $a$ i $b$ sa slike, prethodni slajd
    - $\Delta A = \frac{\Delta t}{P} \cdot A_{ell}$ - II Keplerov zakon
    - Nepoznato $\Delta A$, $\Delta t$, $A_{ell}$
      - $\Delta A$ i $\Delta t$ – sa slike (prethodni slajd, za svaki segment)
      - Nacrtati trougao i odrediti njegovu površinu (ponoviti više puta!)
    - Izračunati masu SMBH
      - III Keplerov zakon
      - $P^2 = \frac{4\pi^2}{G(M+m_{S2})} a^3, M \gg m_{S2}$




| Date (year) | x (arcsec) | dx (arcsec) | y (arcsec) | dy (arcsec) |
|---|---|---|---|---|
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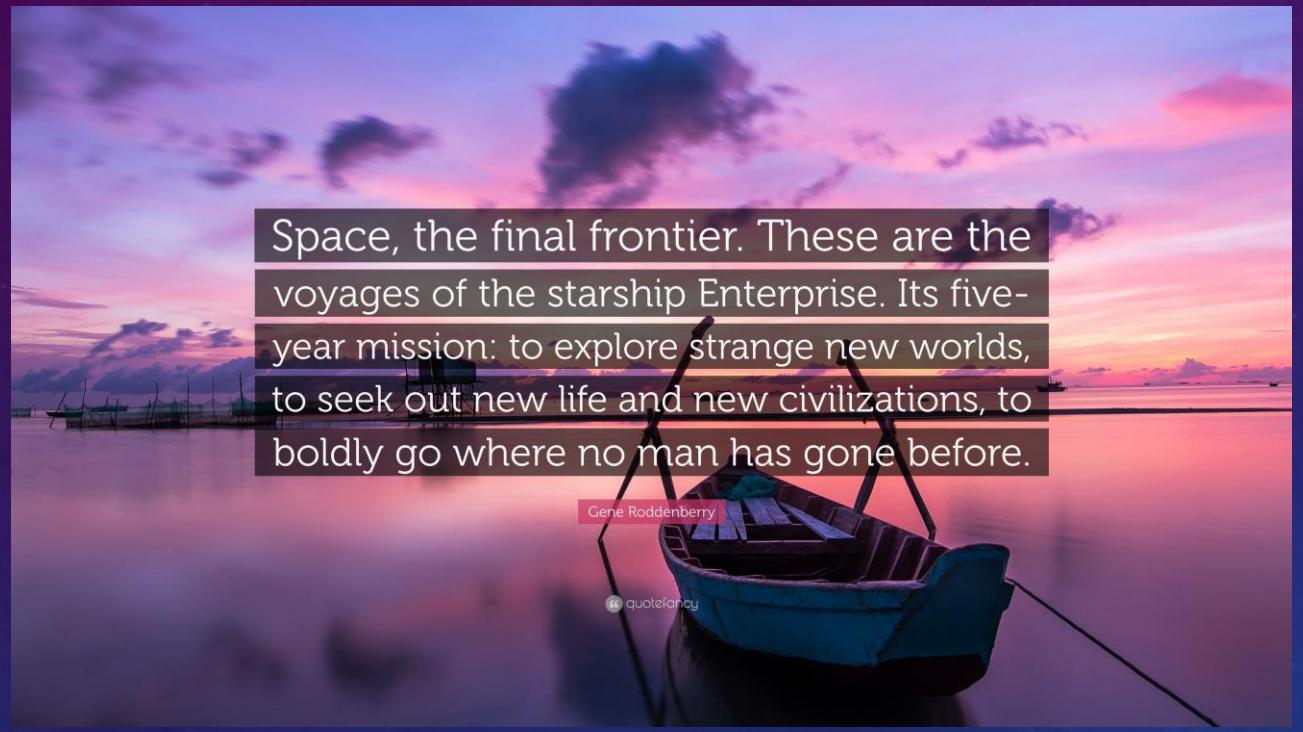
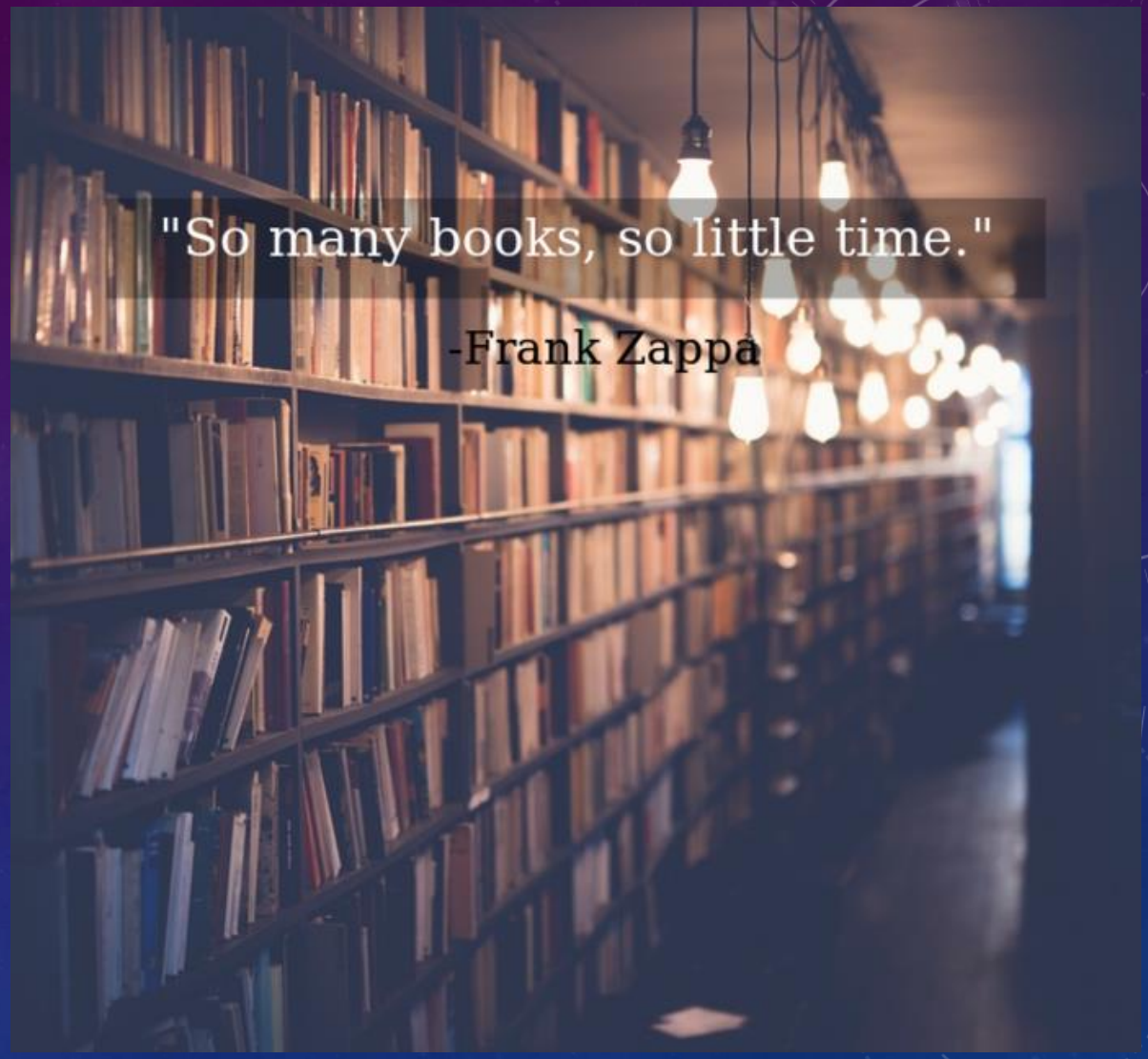
| 1998.365 | 0.036 | 0.001 | -0.167 | 0.002 |
| 1999.465 | 0.022 | 0.004 | -0.156 | 0.006 |
| 2000.474 | -0.000 | 0.002 | -0.103 | 0.003 |
| 2000.523 | -0.013 | 0.003 | -0.113 | 0.004 |
| 2001.502 | -0.026 | 0.002 | -0.068 | 0.003 |
| 2002.252 | -0.013 | 0.005 | 0.003 | 0.007 |
| 2002.334 | -0.007 | 0.003 | 0.016 | 0.004 |
| 2002.408 | 0.009 | 0.003 | 0.023 | 0.005 |
| 2002.575 | 0.032 | 0.002 | 0.016 | 0.003 |
| 2002.650 | 0.037 | 0.002 | 0.009 | 0.003 |
| 2003.214 | 0.072 | 0.001 | -0.024 | 0.002 |
| 2003.353 | 0.077 | 0.002 | -0.030 | 0.002 |
| 2003.454 | 0.081 | 0.002 | -0.036 | 0.002 |

# KORISNI LINKOVI

- SDSS, https://voyages.sdss.org/
- openFITS - Create Images from Raw Data, https://chandra.harvard.edu/photo/openFITS/
- MicroObservatory Robotic Telescope Network
  - https://mo-www.cfa.harvard.edu/MicroObservatory/
  - Observe With NASA, https://mo-www.cfa.harvard.edu/OWN/
  - Search for other planets, https://waps.cfa.harvard.edu/microobservatory/diy/index.php

# NA KRAJU?





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- www.alfa.org.rs